GRAPHICAL ANALYSIS

OF

ROOF TRUSSES;

FOR THE USE OF

ENGINEERS, ARCHITECTS AND BUILDERS,

BY

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ILLUSTRATED BY THREE FOLDING PLATES.

NEW YORK:
JOHN WILEY & SONS,
No. 15 ASTOR PLACE.
1885.

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INTRODUCTION.

The use of Graphical Analysis for the solution of problems in construction has become of late years very wide-spread, and recent discoveries in this line have extended its application in many new directions. The representation to the eye, in one diagram, of the forces which exist in the several parts of a frame, possesses many advantages over their determination by calculation. The accuracy of the figure is readily tested by numerous checks. Any designer who fairly tries the method will be pleased with the simplicity and directness of the analysis, even for frames of apparently complex forms. Those persons who prefer arithmetical computation will find a diagram a useful test of the accuracy of their results. Being founded on principles absolutely correct, these diagrams give results depending for their accuracy on the exactness with which the lines have been drawn, and on the scale by which they are to be measured. With ordinary care the different forces may be obtained much more accurately than the several parts of the frame can be proportioned.

It is advisable to draw the figure of the frame to quite a large scale, as all the other lines are drawn parallel to those of the frame. If it is objected by any that a slight deviation from the exact directions will materially change the lengths of some of the lines, and therefore give erroneous results, it may be suggested that just so much change in the form of the frame will produce this change in the forces; one is therefore warned

where due allowance for such deformation should be made by the proper distribution of material. The comparison of different types of truss for the same locality can be made with ease, and the changes produced in all of the forces in any frame by a modification of a few of its pieces can be readily shown. By applying each new principle to a new form of truss, quite a variety of patterns have been treated without an undue multiplication of figures.

No instruments but those which are at the hand of every draughtsman are needed to apply this method. No claim is made for originality, unless it be in the arrangement of the subjects and some minor details. The graphical determination of bending moments, etc., is added, to make the analysis of the forces complete, and finally some useful data and formulæ are given.

These pages first appeared as a series of articles in "*Engineering News*," and that fact will account for the arrangement of subjects and method of reference.

CHAPTER I.

GENERAL PRINCIPLES.

It is proposed in this series of articles, to explain and illustrate a method for finding the stresses in all the pieces of a roof or other truss, under the action of a steady load, and to show how the wind or any oblique force alters the amount of the stresses arising from the weight. The diagrams, as here developed, are credited in England to Prof. Clerk-Maxwell, and the method is known by his name. But little previous knowledge of Mechanics will be required of the reader to understand the explanations, and we shall endeavor to make all points clear as we proceed.

Taking it for granted that, if two forces, acting at a common point, are represented by the two adjacent sides of a parallelogram *ca* and *cn*, Fig. 2, their resultant will be equal to the diagonal *cb* of the figure, drawn from the same point,—a force equal to this resultant, and in the opposite direction, will balance the first two forces. Thence, considering one half of the parallellogram, we have the well-known proposition that, if three forces act at a single point and are balanced, and a triangle be drawn with sides parallel to the three forces, these sides will be proportional in length, by a definite scale, to these forces. The forces will also be found to act in order round the triangle. If the magnitude of one force is known, the other two can be thus readily determined.

For example:—Let a known weight be suspended from the points 1 and 2, (Fig. 1,) by the chords 1-3, 3-2, and 3-4. Draw *cb* to a convenient scale, vertically, to represent the weight, by so many pounds to the inch. This line will, of course, be parallel to, and will equal the tension on 3-4. Draw *ca*, parallel to 1-3, and *ba* parallel to 3-2. Then will the triangle *cba* represent the forces

which act on the point 3, and they will be found to follow one another round the triangle, as shown by the arrows.

Suppose, next, that a rigid, triangular frame is fastened on the cords of Fig. 1, so that, as shown by Fig. 2, the cords are attached at the vertices of the triangle, while their directions are undisturbed. Then it matters not if the portions of the cords within the triangle be cut away; equilibrium of this combination as a whole will exist so long as the directions of these cords, or forces external to the frame, meet, if prolonged, at a common point. We desire to find the forces, or stresses, exerted in the three pieces of this frame.

Before proceeding further, let us explain the notation which it is proposed to use, and which will be found very convenient when applied to more complex frames. In Fig. 2, write a letter in every space which is cut off from the rest of the figure by lines along which forces act. Thus D represents the space within the triangle, A the space limited by the external forces on 1 and 2, B the space between the pull on 2 and the line which carries the weight. Then let the piece of the frame or the force which lies between any two letters be called by those letters; thus, the upper bar of the triangle is A D, the right hand bar is B D, the cord to the point 1 is A C, that to the weight is C B, etc. In the triangle of forces external to the frame, *c b* will then be the vertical line of force to correspond with C B, *b a* the tension of the cord B A, and lastly *a c* the pull on 1. This method of notation was introduced by Mr. Bow, in his Economics of Construction.

Consider the left hand apex of the triangle. This point is in equilibrium under the action of three forces, viz., A C, C D, and A D; we know the direction and magnitude of A C, and the direction of the other two. The three forces at this joint must therefore be equal to the three sides of a triangle, as before. Notice that the external forces were taken in the order of the letters A C B, as shown by the arrows on the triangle *a c b*. Take then the forces at the left hand angle of the frame in the same order, and commence with the one fully known, or A C. Pass from *a* to *c*, because that is the direction of the action of this force on the joint under consideration. Next, from *c*, draw *c d* parallel to C D, prolonging it until a line from its extremity *d*, parallel to the piece D A, will strike *a*. The direction in which we passed, from *c* to *d*, and

then to *a*, shows that C D and D A both pull on the joint where they meet.

Next take the lowest joint. Remembering again to take the three forces in equilibrium at this joint in the same order as the external forces were taken, and commencing with the first known one, we go, in the stress diagram, from *d* to *c*, because we have just found that the stress on the piece D C is a pull on this joint, next down *c b* in the direction in which the weight acts, and finally we draw, from *b*, *b d* parallel to the piece B D. This last line must close on the point *d*, or the construction has not been carefully made, and the direction in which we draw it, from *b* to *d*, shows that the piece B D exerts tension on the lowest joint. If the reader will run over the triangle for the right hand joint, he will see that the directions just given are complied with.

If we imagine that Fig. 2 is inverted, we shall have three thrusts for the external forces, the stress diagram will be inverted, and there will be compression on each piece of the frame.

In order to make these first principles more plain let us take another case. Suppose a triangular frame, (Fig. 3,) to rest against a wall by one angle, to have a weight suspended from the outer corner, and to be sustained by a cord attached to the third angle and secured to a point 2. Since the frame is at rest under the action of three external forces, and since the known directions of two of these forces, viz., A C and B C, will meet at 4, if prolonged, the thrust of the wall on the frame at 1 must have the direction of the line 1-4, otherwise rotation would take place. These three forces will then be found by the following construction:—Draw *a c* vertically, equal to the known weight acting *down* at the right hand corner; next, from *c*, a line parallel to the cord and force C B, and prolong it until, from its extremity *b*, a line may be drawn, parallel to B A, to strike *a*. As we went from *c* to *b*, and then to *a*, C B must pull on, and B A must thrust against, the frame.

To find the stresses on the pieces:—take whichever joint is most convenient, for instance, the one where the weight is attached; pass down *a c* for the external force, and then, observing the order in which the triangle of external forces was drawn, draw *c d* parallel to C D, and *d a* parallel to D A. C D will be a tie and D A a strut. Take next the joint at 1. Here the reaction, as before ascertained,

is *ba*; next comes *ad*, the thrust of the piece AD against 1; and lastly, *db*, parallel to DB, to close on *b*, the point of beginning, shows that DB also thrusts on the joint 1.

Once more, suppose that a triangular frame, (Fig. 4,) has a weight attached to its lowest angle, and that the other two points are supported by two inclined posts. The forces 1-4 and 2-4 must intersect 3-4 at the same point. Draw *ab* vertically, and equal to the given weight; draw *bc* parallel to 2-4 and *ca* parallel to 1-4. Note that *bc* and *ca* will both be thrusts. Next draw, for the lowest joint, after passing down *ab* for the weight, *bd* parallel to BD, and *da* parallel to DA, thus finding that BD and DA both pull on the joint AB. As in former cases, find *dc*, which is a compressive stress.

Enough examples have been given to show the first principles of this method. Since, in Mechanics, the polygon of forces follows naturally from the triangle of forces, and is simply a combination of several triangles, then, if we have several external forces or a number of pieces meeting at one joint, we follow the same rules. First, draw the polygon of external forces, taking them in regular succession round the frame, either to the left or right, as may seem convenient; next, take any joint and, commencing with the known force or forces, passing over each line in the direction required by the action of the particular force on the joint under consideration, treat the remaining pieces in the order in which the external forces were taken, and draw lines in the stress diagram parallel to these pieces. The direction in which the lines are drawn gives the action of the pieces on the joint. We dwell on this point at some length, because it must be clearly understood to enable one to rightly interpret his diagram.

Referring now to the figures which have been drawn, Prof. Maxwell calls the frame and the stress diagrams *reciprocal* figures, for the external forces, which must meet at one point, (unless parallel) in the frame diagram, make a triangle in the stress diagram, and the pieces which make the triangular frame have their stresses represented by the lines which meet at one point in the stress diagram. This same reciprocity will exist in more complex figures, and it is a valuable check, among many which we have, upon the accuracy of the construction.

Note further, that where the point of intersection of the external forces falls within the triangular frame, the point of intersection of the stresses on that frame, in the stress diagram, falls within the triangle of external forces, and *vice versa.*

CHAPTER II.

TRUSSES UNDER VERTICAL FORCES.

Suppose that the roof represented in Fig. 5 has a certain load per foot over each rafter. Let the whole weight be denoted by W. It is evident that one-half of the load on the rafter C F will be supported by the joint B and one-half by the upper joint; the same will be true for the piece D F; therefore the joint at B will carry ¼ W, the upper joint ½ W, and the joint at E ¼ W. The additional stress produced on C F by the bending action of the load which it carries is not considered at this time, but must be noticed and allowed for separately. Lay off *e d*, or ¼ W, to represent the weight E D acting downward at the joint E, next *d c* equal to ½ W, for the weight D C, and lastly *c b* for the weight at B. Call *e b* the *load line.*

Let the two reactions or supporting forces for the present be considered as a little inclined from the vertical, as shown by the arrows B A and A E. Since the truss is symmetrical and symmetrically loaded, the resultant of the load must pass through the apex of the roof, and, as the two supporting forces must meet this resultant at one point, the two reactions must be equally inclined. Then, to complete the polygon of external forces: — as we have drawn *e d*, *d c* and *c b* in order, passing over the frame to the left, — draw next *b a*, from the extremity *b* of the load line, parallel to the upward reaction B A, and lastly a line *a e*, parallel to the other reaction A E, to close on *e*, the point of beginning.

At the joint B we have equilibrium under the action of four forces, of which the two external ones are known. Therefore, taking them in the same order as in the case of the external forces, and commencing at *c*, pass over *c b* and *b a* in the direction of their actions; then draw *a f* parallel to A F, denoting a pull on the joint B, and prolong

it until *fc*, parallel to F C, will strike *c*, the point of beginning. F C will exert a thrust. Passing next to the apex of the roof, we go down over the line *dc* for the external force, thence up to *f*, and thence draw *fd*, parallel to F D. If this line does not close on *d*, the drawing has not been made with care. The stress *fd* will show thrust on the apex from the rafter F D. As all the stresses are now determined we need not consider the remaining joint.

If the supporting forces had been more inclined from the vertical, the point *a*, of their meeting in the stress diagram, would have been nearer *f*, thus diminishing the tension on A F. The inclination might be so much increased that *a* would fall on *f*, when the piece A F would become unnecessary, the thrust of the rafters being balanced without it. If *a* fell to the right of *f*, *af* would be a thrust. If the two reactions are vertical, as will be the case when the roof truss is simply placed upon the wall, B A and A E, Fig. 6, will each be ½ W, and the point *a* will be therefore found at the middle of *e b*. The polygon of external forces has closed up and become a straight line, but, in the analysis, it must still be used. Thus we have $ed + dc + cb$ for the weights at the joints, and $ba + ae$ for the reactions. The explanation used when the stress diagram of Fig. 5 was constructed will apply equally well to this one.

The reader will find it very useful, when drawing a diagram for himself, to represent different kinds of stress by lines of different colors; thus lines denoting tension may be drawn with red and compression lines with black ink.

The same method of analysis will next be applied to the truss of Fig. 7. Here the rafters are supported at points midway between their extremities. Each point of junction of two or more pieces is considered a joint around which the pieces would be free to turn were they not restrained by their connections with other points. Whatever stiffness the joint may possess from friction between its parts, or from a continuity of a piece, such as a rafter, through the joint, is not taken into account, and may add somewhat to the strength of the truss.

Mr. Unwin in his excellent book on "Iron Bridges and Roofs," treats the rafter as a beam continuous over several supports, and determines the distribution of the load upon the several joints by this hypothesis. But, that such an analysis may be true, it is necessary

that all the points at which the rafter is supported shall remain in a perfectly straight line. As this position cannot be realized, a distribution of the load between any two joints of the rafter proportionally to its distance from each, in accordance with the principle of the lever, will best represent the case. A different distribution of the load, however, if one prefers it, will only require a corresponding division of the load line.

In the truss just taken up, therefore, half of the load on C L will be carried at the lower angle and be represented by the arrow B C; the other half, as well as half the load on D K, will make the force C D, and so on, three of the joints carrying each one quarter of the whole load, and the two others one-eighth each. On a vertical line lay off $gf = \frac{1}{8}$ W, $fe = ed = dc = \frac{1}{4}$ W and $cb = \frac{1}{8}$ W; then, $ba = ag = \frac{1}{2}$ W, the two supporting forces. Now, for the joint B, $cb =$ load, $ba =$ supporting force; then draw al parallel to A L and lc parallel to L C; al is tension and lc compression. At the joint C D, the load is dc, the stress just determined on C L is cl: then draw lk parallel to L K, and kd parallel to K D, to close on d. There is compression on L K and compression on K D. Passing next to the joint D E, ed is the load, dk the thrust of D K on this joint, ki the tension on K I, and ie, to close on e, is the compression on I E. Take next the joint in the middle of the lower tie; we have ik, then kl, and next la, the last of the known stresses. The next piece to which we come is A H; as we have just arrived at a we must pass back horizontally until a line from h parallel to H I will close on i, the point from which we started. The remaining line hf is easily determined by taking either the joint E F or the one near G. It will be noticed that, since the truss is symmetrically made and loaded, the stress diagram is symmetrical; ki must be bisected by al; dk and ei must intersect on al. Attention to such points ensures the accuracy of the drawing.

It is impracticable to determine the stresses at any joint where more than two forces are unknown. In the present case we could not start with the joint C D or with D E; for we should know only the external force or load and have three unknown stresses to find; therefore our quadrilateral, having one side known, might have the other sides of any length, while they were still parallel to the original pieces of the frame. By taking the joints in the order just ob-

served this difficulty was not met with. When, in some cases, we find three or more apparently unknown forces at a joint we may have some knowledge of the proportion which exists between one or more of them and a known force, and can thus determine the proper length of the line in the stress diagram. An example of such a case will be given soon.

We now submit a truss, Fig. 8, which the reader is advised to analyze for himself, as a test whether the principles thus far explained are clearly understood.

The truss represented by Fig. 9 is one well adapted for construction in timber, the verticals alone being made of iron, and it can be used for roofs of large span. In any actual case, before commencing to draw the diagram, assume an approximate value for the weight of the truss, add so much of the weight of the boards and slates, or other covering, as is supported by one truss, and divide this total weight by the number of parts, such as D I or E L, in the two rafters. We thus obtain the weight which is supposed to act at each joint where two pieces of the rafter meet. The weight at each abutment joint will be half as much. If the rafter is not supported at equidistant points, divide the total load by the combined length of both rafters, to obtain the load per foot of rafter, then multiply the load per foot by the distance from the middle of one piece of the rafter to the middle of the next, to obtain the load on the joint which connects them.

Draw the vertical load line equal to the total weight, and, commencing with *b c* as the load on B from one half of C H, space off the weights *c d*, *d e*, etc., in succession, closing at *p* with a half load as at *b*. The point of division *a*, at the middle of *p b*, marks off the two supporting forces *p a* and *a b*, which close the polygon of external forces. Commencing now at B, draw, as heretofore directed, *a b c h a* for this joint. The order of these letters gives the directions of the forces on the joint B. Then for the joint C D we have *h c d i h*; for H K we have *a h i k a*; for D E we have *k i d e l k*, etc. Observe that, by taking the joints in this order, first the one on the rafter, and then the one below it on the tie, we have, in each case, only two unknown forces, out of, at some joints, five forces; and that it is expedient, when possible, first to pass over all the known forces at any joint, taking them in the order observed with the ex-

ternal forces when laying off the load line. The rest of the diagram presents no difficulty. After the stress on N O is obtained, the stress on O G is naturally the same as that on F N, and the diagram will begin to repeat itself inversely. It is therefore unnecessary to draw more than one half of the figure, although it has all been drawn here.

If we observe the kind of stress exerted on the joints by each piece, as we pass over the several polygons, we shall see that the rafter is in compression, as well as all of the inclined pieces, while the horizontal and vertical members are in tension. Sometimes a vertical rod is introduced in the first triangle where the dotted line is drawn. It is evident that this rod will do no work if all the load is assumed to be concentrated on the joints of the rafter, and we can find this out from the stress diagram as well; for, taking the joint below H in the figure, we have three pieces in equilibrium. We therefore begin at *a* and pass to *h* along the stress line lately determined for A H; then we are required to draw a vertical line, and, from its extremity, a horizontal line to close on the point *a* from which we started; the vertical line can therefore have no length. All the work which can ever be done by this vertical rod is to keep the horizontal tie from sagging, by sustaining whatever small weight is found at its foot.

We lately saw, over a large coal shed, a truss intended to be of this type, where the builder, misled probably by the direction of the slope of the main braces of a Howe Truss bridge, had put his timber struts in the other diagonals of most of the trapezoids, their ends being apparently simply abutted at top and bottom. Thanks to a very heavy rafter of one continuous timber, the truss was still in place, but a heavy fall of snow would be likely to ruin it.

The skeleton of a truss which is often built in iron is given in Fig. 10. The loads at the several joints of the rafter are found by the method prescribed for the truss just treated. It will be unnecessary to dwell upon the manner of finding the stresses on the joints B, C D, and H K. The lines of stress for these joints will be, *c h*, *h a*, *h i*, *a k*, *k i*, and *i d*. If then we attempt to analyze the joint D E, we find that, with the external load, we have six forces in equilibrium, of which those along K L, L M, and M E are unknown. If we try the joint L A, we find four forces, three of which are also

unknown. We are therefore obliged to seek some other way of determining one of the stresses.

The external load on the joint E F can be resolved into two components, one parallel to the rafter and the other perpendicular to the same. The component parallel to the rafter must pass down the piece E M, and the transverse component must be balanced by the resistance of the strut M N alone, for the rafter is jointed at E F. This compression on M N must be balanced at its lower end by the equal components, parallel to this strut, of the stresses in the two pieces M L and N O. Therefore the component, in a direction perpendicular to the rafter, of the stress in L M must be one-half of the transverse component of the load on the joint E F. Besides, L M must naturally undergo the same stress with the symmetrieal piece I K, and M N must have the same compression as H I. Any one of these conditions adds enough to the data to enable the diagram for the joint D E to be drawn, and all of them must be fulfilled. Commencing then with *k i*, pass next over *i d*, then *d e*, then draw *e m*, parallel to E M, to such a point *m*, that (having drawn *m l* until its extremity *l* comes in the middle of what will be the space between *e m* and *f n*, or until *m l* equals in length *i k*), the line *l k* shall close on *k* whence we started. The ties and struts can be readily selected by the direction of movement over these lines in reference to the joint D E. The remaining joints when taken in the usual order of succession offer no difficulty, and the other half of the diagram need not be added. The polygon which we have just traced, *k i d e m l k*, affords a good illustration of the rule that the forces which meet at a joint make a closed polygon in the stress diagram, and we may also see that the pieces which surround the space M, for instance, meet at the common point *m* in the diagram. The symmetry of the triangles *h i k* and *m n l*, and their resemblance to *k l o*, are worth noting.

Sometimes it it thought desirable to raise the tie A O, for the purpose of giving more height below the truss and to improve its appearance. The effect of such an alteration is very readily traced, and one can then judge how much change it will be expedient to make. Let it be proposed to raise the tie in the portion A O, and thus to introduce such changes in the other members that they shall take the positions of the dotted lines in Fig. 10, while the load re-

mains the same. The line ah, for the joint B, will then become ah', being prolonged until it meets ch'; next come $h'i'$ and $i'd$; then we easily draw $i'k'$, $k'l'$, $l'm'$, $m'n'$, etc. The struts H I, K L, and M N are the only pieces in this half of the truss unaffected by the change; the amount of increase of the stresses in the others can be readily seen.

To show that it is not necessary to assume that all of the weight is concentrated on the rafters, let us discuss the truss in Fig. 11, supposing that the joints Q R and R S carry their share of the weight of the pieces which touch these joints. Each supporting force will still be one-half of the total load, but the two will no longer equally divide the load line between them, nor can the load line be at once measured off as equal to the total weight. Commencing, if convenient, with the extremity H of the truss, lay off hi, ik, kl, etc., ending with op; lay off next the reaction, pq, equal to one-half the total weight, acting upward, then qr and rs downward, and finally sh upward, for the other supporting force, to close on h. The polygon of external forces, therefore, doubles back on itself, as it were, and hp is still only the load on the rafters. The diagram can now be drawn, taking three joints on the rafter in succession before trying the joint Q R, and, when taking that joint, remember that there is a load upon it. The loads on the horizontal tie cause the stresses on its three parts to be drawn on three separate lines, instead of being superimposed as in the figures before given.

CHAPTER III.

TRUSSES WITH HORIZONTAL THRUST.

When it is desired to strengthen the rafters in a roof of moderate span by supporting them at their middle points, a simple means, often employed, is to spike on a piece from the lower end of one rafter to the middle of the other, as shown in Fig. 12. The two pieces may or may not be fastened together where they cross. At the first glance we should say that, to draw the diagram, we must lay off the load line *k e*, divide it as usual, then, commencing at the joint E, draw from *f* and *a* lines parallel to F B and B A, and prolong them (see the dotted lines) until they meet at *b′*. Next, for the joint F G, we should get the lines *b′c′* and *c′g*. For the apex we should have three lines, viz., *h g*, *g c′*, and a line from *c′* parallel to C H to strike *h*. There is evidently something wrong here. If we start from the other point of support K, we shall obtain the remainder of the diagram in dotted lines, and find that we have two points marked *c′*, some distance apart, which ought to come together; we also have two conspiring forces, *g c′*, and *h c′*, whose vertical components ought to balance *h g*.

Abandoning this diagram for the present, let us start at the apex of the roof, where we may feel sure that there are but three forces, one of which is known. Taking the load *h g* at that point, draw the full lines *g c* and *c h*. Next for the joint G F, starting with *c g*, pass down *g f* and draw *f b* and *b c*. The joint H I will similarly give the figure *i h c d i*. Lastly, the joint A C will add *b a* and *a d* to the stresses *d c* and *c b*. Here again we find two points *a* widely separated. To make equilibrium for the joint E, therefore, as the diagram now stands, we have *a b*, *b f*, *f e*, *e a*, and a force *a a* required to close this polygon. This force can be no other than the horizontal thrust which must be supplied to keep this truss from

sliding outwards on the wall-plates, on the supposition that the points of meeting of two or more pieces are true joints (ones about which the parts are free to turn). This horizontal reaction may come from the wall, or a tie rod from E to K will accomplish the same end. The pieces of this truss are all in compression. That the truss is not in equilibrium without this resistance at E and K is seen, if we suppose that E and K are not prevented from sliding laterally; the joint A C will drop, the joints F G and H I will approach one another, and the apex will rise. This change will take place unless some restraining force is applied. The trouble arises from the existence of the four sided figure surrounding the space C, which figure is free to change its form. A tie from the apex to the joint C A would prevent the movenent, and it would supply the missing line $c'c'$ in the dotted diagram. In that case it is well to note that the stress is greater in most of the pieces than when the thrust comes from the wall. A strut from the joint F G to the joint H I would also make the truss secure; the reader can try the diagram with that change for himself, and see what piece has its stress reversed by the change. The supporting force at E might now rather be represented by a line from e upward to the right to a, than by two rectangular components; and similarly for the one at K.

As these trusses are usually made, reliance against change of form, when no horizontal thrust is supplied, is placed upon the stiffness of the rafters, which are of one piece from ridge to eaves, and on that of the two braces. The bending moment at the joints where movement would take place comes from this otherwise unbalanced horizontal thrust at the wall plates. Bending moments on a rafter will be considered later. It will be noticed that cd equals ba, or that the thrust is constant all along the brace. This would naturally be expected; for the component of the load at the joint H I which starts down the brace will pass to E without being affected by crossing the other brace; and if it were not for the tendency to rotate, spoken of above, spiking the two braces together would be needless, were it not for the reason that the braces are better able to resist thrust by mutually staying one another.

A consideration of the trapezoidal truss, represented by Fig. 13, will bring out two or three points which will be of use in the anal-

ysis of other trusses. In this case, let us suppose the load to be on the lower part, or bottom chord, of the truss; in order to separate the supporting forces from the small weights on the ends of the truss, and to enable the latter to come consecutively with the other weights in the load line, let us draw the supporting forces above the tie, instead of below as before. The rectangle formed by the two vertical and two horizontal pieces might become distorted; we will therefore introduce the brace H I, represented by the full line. The dotted line shows a piece which might have been introduced in its place. The rectangle is thus divided into two triangles and movement prevented.

If the truss is symmetrically loaded, or C D = D E, we shall get the first stress diagram. The stress on each vertical is equal to the load at its foot, and, if the load had been on the upper joints, no stress would have been found on the verticals. It will also be seen that the piece H I has no stress upon it. (If the reader does not clearly see this point, refer to what was said, on page 16, about the dotted line in Fig. 9.) It is therefore evident that a trapezoidal, or queen-post truss, when symmetrically loaded, requires no interior bracing. This fact might be readily seen if we considered the form assumed by a cord, suspended from two points on a level, and carrying two equal weights symmetrically placed.

The second stress diagram will be drawn when the weight on the joint C D is less than that on D E. Let us suppose that $b\,c$ and $e\,f$ are of the same magnitude as in the first diagram, and let the span of the truss, or distance between supports, which we shall denote by l, be divided by the joints into three equal parts. The first step is to find the supporting forces. Since equilibrium exists, if each external force be multiplied by the perpendicular distance of its line of action from any one assumed point, which distance may be called its leverage, and all the products be added together, those which tend to produce rotation about this point in one direction being called plus, and those tending the other way minus, it is necessary that the sum of these products shall be zero; otherwise the rotation can take place. A convenient point to which to measure the distances will be one of the points of support, for instance, the right hand one. Then we shall have

$$AF.l - FE.l - ED.\tfrac{2}{3}l - DC.\tfrac{1}{3}l - CB.0 + BA.0 = 0; \text{ or}$$

$$AF.l = FE.l + ED.\tfrac{2}{3}l + DC.\tfrac{1}{3}l \quad \text{therefore}$$

$$AF = FE + \tfrac{2}{3}ED + \tfrac{1}{3}DC.$$

If D E be taken as 3 C D,

$$AF = FE + \tfrac{7}{9}ED.$$

It is plain that the object of taking the point or axis at B is to eliminate B A, and have only one unknown quantity, A F. Lay off this value at *f a* and then *a b* equals the remainder of the load. This method of determination is called ***taking moments***, and is at once the simplest and most generally applicable.

The point *a* being thus located, we can proceed to draw the second diagram. The construction requires no explanation, but we will simply call attention to the fact that a compressive stress is now found to exist in H I. If, in place of the diagonal represented by the full line, the one shown by the dotted line is now supplied, the reader can without difficulty trace out for himself the change in the diagram which is denoted by the dotted lines, and the letters marked by accents. The stress on this diagonal will be seen to be tensile, while it is worthy of notice that the only pieces affected by the substitution of one diagonal for the other are those which form the quadrilateral enclosing the diagonals.

If, at another time, this excess of load might fall on D C in place of E D, it is evident that the stress on either diagonal would be reversed. Now, as a tension diagonal is likely to be a slender iron rod, which is of no practical value to resist a thrust, while the compression member, unless made fast at its extremities, will not transmit tension, a weight or force which may be shifted from one joint to another may require the designer to have two diagonals in the same rectangle or trapezium, or else to so proportion and fasten one diagonal as to withstand either kind of stress. Where both diagonals occur the diagram can still be drawn. Determine which kind of stress, tension or compression, the two shall be designed to resist, and then, on drawing a diagram, and arriving at a particular panel or quadrilateral, try to proceed as if only one of the diagonals existed. If a contrary stress to the one desired is found to be needed, erase the lines for this panel only, and take the other diagonal. In the treatment for wind, yet to be taken up, this method must often be used, since the wind may blow on either side of the roof. This truss can be used for a bridge of short span.

Another example where the horizontal thrust of the truss on the walls must be ascertained will now be given. The frame drawn in Fig. 14 is called a hammer beam truss, and is a very handsome type often employed, in this country and abroad, for the support of church roofs, the braces being visible from below, and the spaces containing more or less ornamental work. When the church has a clear-story, the windows come between the trusses at B, the truss is supported on columns, and the roof of the side aisle takes up the horizontal thrust. If there are no side roofs, the main walls are properly strengthened by buttresses.

It will first be well to notice the fact that a curved piece in a truss, so far as the transmission of the force from one joint to another is concerned, acts as if it lay in the straight line between the two joints. The curved members in the present example are the quadrants of a circle. They may have any other desired curve, depending somewhat upon the pitch of the roof. If, now, we consider the point of support B P of the truss, and remember that the curved piece A O transmits the force between its two extremities as if it were straight, it will be evident that the thrust of this inclined piece must have a horizontal component which cannot be neutralized by a vertical supporting force alone. Therefore, in addition to the reaction of half the weight of the roof and truss, there must be supplied by the wall, assisted perhaps by a buttress or a side roof, a certain horizontal thrust.

To determine the value of this thrust: — Let W equal the weight of truss and load. We have nine loaded joints, and there is, therefore, $\frac{1}{8}$ W at each joint except the two extreme ones. Draw the dotted line from 4 to the joint 2, and the similar one on the other side. The whole truss may then be considered to be made up of a small triangular truss 1 2 3, carried upon a trapezoidal truss 4 2 3 5, the brace 4-2 being made up of an assemblage of pieces. As the load is symmetrically distributed, the trapezoid will require no bracing, (compare Fig. 13,) and L A will have no stress upon it. L A will be required when the wind acts upon the roof. Considering then the trapezoidal truss 4 2 3 5 alone, the joint 2 will carry a load equal in amount to that on D M, E K, and F I, or $\frac{3}{8}$ W, the joint 3 will carry the same amount, while 4 will support $\frac{1}{8}$ W from C N, and 5 the remainder. If, then, we lay off on a vertical line $\frac{3}{8}$ W, for

the load on 2, and draw lines parallel to 2-4 and 2-3 from its extremities, the line parallel to 2-3 will be the stress in the same, and will also, since the load is vertical, be the horizontal thrust of the foot of the compound brace 2-4. This force is marked H in the dotted triangle drawn below the truss. A reference to the trapezoid, Fig. 13, will aid one, perhaps, in understanding the above.

We have now the data for the stress diagram, of which one half only is here given. For the point 4, or B P, we have the upward supporting force *b p*, = ½ W, next *p a*, the horizontal thrust of the wall, etc., against the joint, *a o* parallel to the line of action of A O, and finally *o b*, the pressure of the post O B on 4. The resultant of *b p* and *p a*, or *b a*, may of course be used for the reaction of the wall. Taking next the joint 6, we have *c b*, the load, *b o* the thrust of B O, and we then draw *o n* and *n c*. The joint C D gives *d c n m d*. The joint M A already has the lines *m n*, *n o*, and *o a*; since the line which is to close on *m* must be parallel to L M, and *a* is already vertically over *m*, *a l* can have no length, and there is no stress on A L. Upon taking the joint D E we find also that no stress exists on L K. The reader must not think this fact at variance with the value H which was said to exist in 2-3 when we considered the trapezoid alone; the triangular truss 1 2 3 will plainly cause a tension in 2-3, and, with this distribution of load, such tension will exactly neutralize the compression caused in the same piece by 4-2. If one will consider the truss as loaded at 6, 2, 1, 3, and 7 only, thus doing away with N M, K I, I G, etc., he will find that a diagram will then give him some compression on K L.

CHAPTER IV.

TRUSSES FOR FLAT ROOFS.

It is sometimes the case that, in covering a large public building, it is desirable to have the interior clear from columns or partitions, while a roof of very slight pitch is all that is needed. Also, in a public hall, galleries may be suspended from the roof, and the weight of a heavily paneled or otherwise ornamented ceiling may be added to what the truss is ordinarily expected to carry. The trusses are then commonly made of timber, some of the parts in tension being iron rods. As it is not expedient to have a truss of much depth, one of several types of parallel chord bridge trusses may be employed, for instance, the "Warren Girder," of Fig. 15, which is an assemblage of isosceles, often of equilateral triangles. The depth might well be made less than here drawn, but, for clearness of figure, we have not made the truss shallow.

If the roof pitches both ways from the middle of the span, the top chord may conform to the slope, making the truss deeper at the middle than at the ends; but a light frame may be placed above, as shown by the dotted lines, and supported at each joint of the top chord, when we shall have the practical advantage that all the braces will be of the same length and the truss will be easily framed. If the roof pitches slightly transversely to the trusses, it will be convenient to make them all of the same depth and put on some upper works to give the proper slope. The ends of the truss could readily be adapted to a mansard roof.

We have taken a case where each top joint is supposed to be loaded with the weight of its share of roof, in which case the joint L M or P Q will have three-quarters of the weight on N O or O P, and where the bottom joints carry the weight of the ceiling, and in addition the tension of a suspending rod to a gallery on each side.

The load line will be equal to the weight on the upper part of the truss, and the polygon of external forces will overlap, as in Fig. 11, previously explained. We go from *r* to *k*, for the load on the top joints in sequence, then up to *w*, then down to *s* for the load on the bottom joints, and finally up to *r* again. Upon drawing the diagram it will be seen that the stress is compression in the top chord and tension in the bottom chord; that the stresses in the chords increase from the supports to the middle; that the stresses in the braces decrease from the ends of the truss to the middle, and that alternate ones are in compression and tension, those which slant up from the abutments toward the centre being compressed, and those which incline in the other direction being extended. The tie-braces are, therefore, A B, C D, F G, and H I. A decrease of depth in the truss will increase the stresses in the chords.

Another truss with parallel chords may also be employed, in which the braces are alternately vertical and inclined. The designer will choose whether the verticals shall be ties and the diagonals struts, in which case the type is called the "Howe Truss," or the verticals struts and the diagonals ties, when the truss may be called the "Pratt," or perhaps the "Whipple." There is an advantage in having the struts as short as possible, but, if one desires to use but little iron, the "Howe" is a good form.

To decide which diagonal of the rectangle shall be occupied by the piece: — Start from the wall as a fixed point; it is evident that, to keep the load C D from sinking, C Q must be a strut. If we wish to put a tie in this panel, it must lie in the other diagonal, shown by the dotted line. C D now being held in place, P O as a strut will uphold D E. We thus work out from each wall until we have passed as much load as equals the amount supported, or the reaction, at that wall. If the last load passed exactly completes the amount required to equal the reaction, no diagonal will be required in the next panel. As explained before, we might draw diagonals, one in each panel, sloping in either direction as we pleased, and then construct the stress diagram. If we found a stress on any diagonal opposite to the stress we desired, we could then erase that diagonal and substitute the other, erasing also so much of the diagram as referred to the pieces of that panel. It is better, however, to draw the load line first, find the dividing point *a* for the two reactions, see

what load it cuts, and then incline all the braces similarly from each wall towards that load. In the present example C D is supposed to be four times D E, etc. A tower on that end of the truss or some suspended load will account for the difference. It is sometimes desirable to find the position of the resultant of the load, often called its centre of gravity. This point is the one at which the total load on the truss might be considered to be concentrated, so far as the pressure on each wall is concerned. If we recall the manner in which the supporting forces were determined when the load was unsymmetrical, we can see that, to find the position of the resultant of the separate loads, we must multiply each load by its horizontal distance from some convenient point, for instance a point of support, and divide the sum of these products by the total load; the quotient will be the desired distance of the resultant from the same point. As the panels are all equal, a panel may be taken as a unit of distance; call a panel length p, and the ordinary weight on a joint, w. Then we shall have, taking moments about H,

$$\frac{w . p\,(1+2+3)+4w . 4p}{7w}=3\frac{1}{7} \text{ panels,}$$

the distance of the resultant from the end H. The loads directly supported at the walls do not affect the truss. If the resultant divides the truss in the proportion of $3\frac{1}{7}$ panels to $1\frac{6}{7}$ panels, the two supporting forces, excluding the amounts to balance B C and G H, will be in the same proportion, the larger force being on the side of the shorter division. Therefore, drawing the load line $b\,h$, and measuring off $b\,c$ and $g\,h$, divide cg at a into $\frac{22}{35}$ and $\frac{13}{35}$. The two supporting forces will then be $b\,a$ and $a\,h$. Draw the stress diagram as usual; the diagonals will all come in compression as intended. There will plainly be no stress in the dotted vertical O N. The stress on the chords being inversely proportional to the depth of the truss, economy of material in the chords will be served by making the depth as much as possible. In bridge trusses this depth is seldom less than one-eighth of the span.

If the joint D E also was heavily loaded, we could draw another diagram for that case, and, as the joints in succession had their loads increased, we might make as many diagrams. From a collection of diagrams for all positions of a moving load, we could select the max-

imum stress on each piece. A truss designed to resist such stresses would then answer for a bridge. We should find that the greatest stresses in the chords occurred at all points when the bridge was heavily loaded throughout, and that the greatest stress in a diagonal occurred when the bridge was heavily loaded from this piece to one end only, that end generally being the more distant one. As we have more expeditious methods of treating a bridge truss, this one is not used. The reader who is interested in the graphical treatment of such a subject can find it explained in "Graphical Analysis of Bridge Trusses." *

* Graphical Method for the Analysis of Bridge Trusses, extended to Continuous Girders and Draw Spans, by C. E. Greene. New York, D. Van Nostrand, 1875.

CHAPTER V.

FORCES NOT APPLIED AT JOINTS.

We have heretofore treated all the trusses as if the loads were concentrated at those points only which were directly supported. It not seldom happens that the cross-beams or purlins, which connect the trusses and convey the weight from the secondary rafters to the main rafters, rest at points intermediate between the joints. Let us in the present example, Fig. 17, assume that a load rests upon the middle of each of the upper rafters. If we pursue the previous method (leaving out of account, for the present, the bending action of this load), one half of the load E G will be supported at each of the joints C E and G K, and similarly for the load K M. Therefore, having laid off the weights and the two equal reactions of the walls on the load line of the first diagram, we may increase the loads on the joints C E, G K and M O, by the new points of division, and complete this diagram, taking first B, then the next joint on the inside, and then the outside one. It will be noticed that all of the pieces except the rafters are ties.

This diagram gives but one stress along the whole of the upper rafter; but it is plain that the vertical load E G must have a component along the rafter which will cause a different stress to exist on E T from what exists on G T. If, however, we suppose a joint at E G, the transverse component will cause it to yield, as there is no brace beneath to hold it in place. Let us supply an imaginary force E F, shown by the dotted line, equal and directly opposed to this transverse component, which will take the place of a perpendicular strut, will steady the joint, and will leave the longitudinal component to affect the rafter. But the transverse component of F G actually gives a pressure at the joints C E and G K, while the imaginary force E F, just added, will lift the ends of this rafter by the

same amount; therefore we must restore the pressure by adding imaginary forces, each one-half of E F, at C D and G H. This added system of forces cannot interfere with the stresses in any other pieces, for they balance by themselves. Treat the similar load K M in the same way.

Draw now, for the second diagram, *p a* and *a b*, the two supporting forces, each equal to one half the total load. Then lay off *b c*, as before; draw the dotted line *c d*, equal and parallel to the first imaginary force C D, then *d e*, vertical as before, then *e f*, equal to, and in the direction of E F, then *f g*, and so on, arriving finally at *p*, as usual.

The construction of the rest of the diagram presents no difficulty; the joints are taken in the same order as before, and, when we have more than one external force on a joint, we take them in succession, passing in the order first observed for the external forces. When we reach the upper rafters, it will be noticed that *g* falls on the line *e t*; *e t* will be seen to be greater and *g t* less than the line for the same piece in the other figure. The first diagram gives the stress which would exist in the whole length of the rafter E T G if the load F G were actually at its extremities; but, being at its middle point, one-half of the longitudinal component of F G goes to diminish the compression otherwise existing in G T, and the other half to increase the compression in E T. A comparison of the two diagrams will show the truth of the former statement, that the system of imaginary forces does not affect any of the truss outside of the particular pieces to which it may be applied. It is still necessary to provide for the bending action of the transverse portion of F G, or a force equal and opposite to E F, upon the rafter, considered as a beam extending from hip to apex, a joint of course not being made at E G. This subject will be taken up later.

The treatment of loads or of forces not directly resisted, as above, is given by Mr. Bow in his "Economics of Construction," and may be applied to frames where one or more of the internal spaces are not triangles, but quadrilaterals; the truss would then be liable to distortion, unless the resistance of some of the pieces to bending or the stiffness of the *theoretical* joints was called into play. A too frequent use of this treatment in the same diagram will, however, be apt to make confusion.

CHAPTER VI.

WIND PRESSURE.

All of the forces hitherto considered have been vertical; but when we treat of the action of the wind such is no longer the case. It was formerly usual to deal with the wind as a vertical load, added to the weight of the roof, snow, etc., and the stresses were then obtained for the aggregate pressure. Tredgold recommended 40 pounds per horizontal square foot as a fit maximum of wind force to be provided for. This treatment manifestly cannot be correct, even if the added vertical force were placed on one side of the roof only.

The wind may be taken without error as blowing in a horizontal direction; it exerts its greatest pressure when blowing from a point of the compass at right angles to the side of a building; it consequently acts upon but one side of the roof, loads the truss unsymmetrically, and sometimes causes stresses of an opposite kind, in parts of the frame, from those due to the steady load. Braces which are inactive under the latter weight may therefore be necessary to resist the force of the wind.

It will not be right to design the roof to sustain the whole force of the wind, considered as horizontal, nor will it be correct to decompose this horizontal force into two rectangular components, one perpendicular to the roof, and the other along its surface, and then take the perpendicular or normal component as the one to be considered; for the pressure of the wind arises from the impact of particles of air moving with a certain velocity, and those particles are not arrested, but only deviated from their former direction upon striking the roof. Nor will the analysis of a jet of water striking an inclined surface answer here, for water escapes laterally against a comparatively unresisting medium, the air, while the wind particles,

deflected by the roof, are turned off into a stream of similar air, also in motion, which affects their lateral progress. We are obliged therefore to have recourse to experiments for our data, and from them to deduce a formula. It appears that, for a given pressure per square foot against a vertical plane, the pressure exerted by a horizontal wind current, against a plane inclined to its direction, is perpendicular to its surface, and is greater than the normal component of the given horizontal pressure. Unwin quotes Hutton's experiments as showing that, if P equal the horizontal force of the wind on a square foot of a vertical plane, the perpendicular pressure on a square foot of a roof surface inclined at an angle i to the horizon, may be expressed by the empirical formula

$$P \sin. i^{1.84 \cos. i - 1}$$

If, then, the maximum force of the wind be taken as 40 pounds on the square foot, the perpendicular or normal pressure per square foot on surfaces inclined at different angles to the horizon will be:

Angle of Roof.	Normal Pressure.	Angle of Roof.	Normal Pressure.
5°	5.2 lbs,	35°	30.1 lbs.
10	9.6	40	33.4
15	14.0	45	36.1
20	18.3	50	38.1
25	22.5	55	39.6.
30	26.5	60	40.0

For steeper pitches the pressure may be taken as 40 pounds.

Some experiments of our own, with a section of roof, so far as they have been at present carried, tend to confirm the above values. In the case of a shingled roof, under the heavier pressures a very slight force in the plane of the roof was manifested, which might be attributed to the pressure of the wind against the butts of the shingles. It was too slight to be of any consequence, and on a smoother surface it would seem that the component parallel to the roof, arising from the friction of the air as it passed up along the surface, would be practically inappreciable.

Although somewhat higher pressures have very rarely been registered, it will be sufficient to provide against a horizontal force of wind of 40 pounds on the square foot, and to take from the table the pressure per square foot of roof surface for that side of the roof on which the wind blows, which pressure will be exerted normally to

the surface and will be the only pressure on the truss from wind at that time.

The truss of Fig. 18 is supposed to be under the action of a wind from the left. If the truss is 67 ft. span, and height 15 ft., the angle of inclination will be $24° 7'$, and the normal pressure, interpolated from the table, will be 21.8 pounds. The rafter will be 36.7 feet long. If the trusses are 10 feet apart, the normal wind pressure on one side will be

$$36.7 \times 21.8 \times 10 = 8000.6 \text{ pounds.}$$

For steady load of slates, thin boards, rafters, purlins, and truss, let us assume 11 pounds per square foot of roof, or

$$11 \times 36.7 \times 2 \times 10 = 8074 \text{ pounds, total vertical load.}$$

We will, in the present case, treat the two kinds of external force separately. The diagram on the right for steady load needs no description. The supporting forces will each be 4037 pounds, and the weight on the joints of the rafters will be, $672\frac{5}{6}$ pounds for the end ones, and $1345\frac{2}{3}$ pounds for the others. The truss is drawn to a scale of 40 feet to an inch, and both diagrams are drawn to a scale of 8000 pounds to an inch. The above weights are laid off on the vertical load line and the diagram then drawn. In actual practice the diagram should be much larger, say 1000 pounds or 800 pounds to an inch. The stresses on the various pieces for half of the truss are given in the table below, the sign + denoting compression, and the sign –, tension.

We come next to the wind diagram. The normal pressure of 8000 pounds distributed uniformly over the whole of the left side of the roof will have its resultant, shown by the dotted arrow, at the middle of the length of the rafter. To find the supporting force on the right we may take moments about the left hand wall, remembering to multiply each force by the lever arm drawn perpendicular to its direction: or

$$AP \times HR = 8000 \times HK,$$

or $$AP \times 61.15 = 8000 \times 18.35;$$

whence $AP = 2401$ pounds, and $AH = 5599$ pounds.

But, since these arms, HR and HK, are proportional to the span and the part of the horizontal tie cut off by the resultant, an easier way to get the supporting pressures due to an inclined force is to

prolong this force until it cuts the tie, when the two reactions will be proportional to the two segments into which the tie is divided, the larger force being on the side of the shorter segment, or the side on which the wind blows.

The pressures on the joints will be 2666⅔ pounds each on I K and K L, and 1333⅓ pounds on H I and L M, as denoted by the arrows. Draw now *m h*, by scale, equal to 8000 pounds, so inclined as to be perpendicular to the roof; divide off the reactions of the supports by means of the point *a*, and lay off the joint forces in succession, *m l*, *l k*, *k i*, and *i h*. Thus we complete the polygon of external forces, always drawing the load or pressure lines parallel to the forces.

Proceed with the construction of the diagram by the usual rules, remembering that wind alone is being treated. After the joint K L has given *l k c d e l*, the joint E A gives *e d a f e*. Taking next the apex L M, and passing along *m l*, *l e*, and *e f*, we find that there will be no line parallel to F G, since *g m*, parallel to G M, will exactly close on *m*, the point of beginning. As no stress passes through F G, the remainder of the *bracing* on this side can experience no strain, and therefore the compression *m g* affects the whole of the right hand rafter, while the tension *f a* affects the remainder of the horizontal tie. To appreciate how this can be, imagine all of the braces in the right half to be removed; it is evident that the right rafter is a sufficient support to the joint L M, conveying to the wall the stress *m g* which compresses its upper end, while the tie F A keeps the truss from spreading. The stress triangle for the point P will therefore be *m g a m*.

At another time the wind may blow on the right side. Then the braces on the right will be strained as those on the left are now, and those on the left will be unstrained. The wind stresses are placed in the third column of the table. As in this truss they are all of the same kind as those from the vertical load, they are added together to give the total or maximum stress. The force *m g*, being smaller than, while it is of the same kind as *l e*, is of no consequence; for, with wind on the right, M G would have to resist a stress equal to *l e*.

A combination of the two supporting forces at each end, as shown in the figure, by either the parallelogram or triangle of force, will

give the amount and direction of each reaction from the combined load. Wind on the other side will exactly reverse the amounts and bring them on the opposite side of the vertical line.

TABLE OF STRESSES FOR FIG. 18

PIECE.		Steady Load.	Wind.	Total.
		lbs.	lbs.	lbs.
Tie.......	A B ..	−7540	10440	17,980
	A D ..	−6200	7160	13,360
	A F ..	−4520	3900	8,420
Braces....	E F ..	−1830	3990	5,820
	C D ..	−1500	3280	4,780
	B C ..	+1230	2667	3,900
	D E ..	+1840	4000	5,840
Rafter.....	I B...	+8280	9530	17,810
	K C ..	+7690	9530	17,220
	L E ..	+5760	6550	12,310

If the truss is simply placed upon the wall-plates, and either of the supporting forces makes a greater angle with the vertical than the angle of repose between the two surfaces, the truss should be bolted down to the wall; otherwise there will be a tendency to slide, diminishing the tension in the tie, perhaps causing compression in that member, and changing the action of the other parts of the truss. This matter will be treated of further.

If the weight of snow is also to be provided for, it may readily be done by taking the proper fraction of the stresses from the steady load and adding them to the above table.

We propose, in the example illustrated by Fig. 19, to consider the truss as supported on a rocker or rollers at the end T, where the small circle is drawn, to allow for the expansion or contraction of the iron frame from changes of temperature. It is therefore plain that the reaction at T must always be vertical. The truss is supposed to be 79 feet 8 inches in span, and 23 feet in height, which gives an angle of 30° with the horizon, and makes the length of rafter 46 feet. It would be proper usually to support the rafter at more numerous points; but our diagram would not then be so clear, with its small scale, from multiplicity of lines, and one can readily extend the method to a truss of more pieces.

This frame supports 8 ft. of roof, and the steady load per square foot of roof is taken, including everything, as 14 lbs. The total vertical load will then be

$$14 \times 46 \times 2 \times 8 = 10304 \text{ lbs.},$$

or $1717\frac{1}{3}$ lbs. on all joints except the extreme ones.

We find, from the table given on page 32, that the normal pressure of the wind, for a horizontal force of 40 lbs. on the square foot, may be taken as 26.4 lbs. per square foot of a roof surface inclined at an angle of 30°. The total wind pressure, normal to the roof, will therefore be

$$26.4 \times 46 \times 8 = 9715.2 \text{ lbs.},$$

or 3238.4 lbs. and 1619.2 lbs. on the middle and end joints respectively of one rafter. The truss is drawn to a scale of 50 feet to an inch, and the diagram to that of 10,000 lbs. to an inch.

The diagram for steady load is the middle one, and a little more than one half is drawn. The only piece at all troublesome is G F. On arriving in our analysis at the apex of the roof, or at the point directly over A in the tie, we find three pieces whose stresses are undetermined ; but, as we have reached the middle of the truss, we know that the diagram will be symmetrical, and therefore that gf will be bisected by $a\,l$. In the case of an unsymmetrical load we can recommence at the other point of support and close on the apex. The stresses caused by this load are given in the first column of figures in the table on page 38, compression being marked +, and tension —. If snow is to be provided for, make another column in the table, of amounts properly proportioned to those just found.

Upon turning our attention to the other diagrams, we shall find that the rollers at T cause something more than a reversion of diagram, often a considerable variation of stress, when the wind is on different sides of the roof. Taking the wind as blowing from the left, we draw the diagram marked **W. L.** The line qm, 9715.2 lbs., is divided and lettered as shown for the four loads at the joints where the arrows are drawn. The resultant of the wind pressure, at the middle point of the rafter, when prolonged by the dotted arrow, will divide the horizontal line or span in the proportion in which the load line should be divided to give the two parallel reactions, if there were no rollers at T. This proportion, with a pitch

of 30°, is 2 to 1; it locates the point a', and gives $m\,a' = 6476.8$ lbs., and $a'q = 3238.4$ lbs.

But the reaction at T must be vertical, and consequently the vertical component of $a'q$ will be found at T, while the horizontal component of $a'q$ must come, through the horizontal member, from the resistance of the other wall. Therefore draw $a'a$ horizontally, and we shall get $a\,q$ as the reaction at T, while $m\,a$, to close this triangle of external forces, must give the direction and amount of the reaction at M.

It may at first sight strike the reader that this analysis will not be correct, for, if only the vertical component is resisted at T, and if we decompose the resultant of the wind pressure at O, where it strikes the roof, into two components, we get results as follows:

Vertical component of 9715.2 lbs., for angle 30° = 8413.65 lbs.
Horizontal " " " = 4857.6 "

The vertical from the middle point of the rafter will divide the span at ¼ M T. Therefore, amount of vertical component carried at T=2103.4 lbs.; remainder supported at M with all of horizontal component. So far, correct. Take next into account the tendency of the horizontal component at O to cause the truss to overturn. It naturally decreases the pressure at M and increases that at T, or, in other words, the couple formed by the product of the horizontal component at O and half the height of truss must be balanced by a tension at M and a compression at T with a leverage of the span. Making the computation of this tension or compression, T, we have

$$4857.6 \times 11.5 = T \times 79\tfrac{2}{3}, \text{ or } T = 701.2 \text{ lbs.}$$

$$2103.4 + 701.2 = 2804.6 \text{ lbs.} = \tfrac{1}{3} \text{ of } 8413.7$$

as obtained by the first process.

Still another way to find the supporting forces is, to prolong the resultant until it intersects the vertical through T, then to draw a line from M to the point of intersection, and finally to draw $m\,a$ and $q\,a$ parallel to the lines from M and T. This method depends for its truth on the fact that the external forces which keep the truss in equilibrium, not being parallel, must meet in one point.

Having completed the triangle of external forces, and laid off the pressures at the joints, we can readily draw the diagram. It will be found, as in Fig. 18, that braces on the right experience no strain, the lines $g\,f$ and $e\,q$ closing the polygon which relates to the joint

PQ. If the lower tie were cambered, or inclined from both ends to the middle, stresses from wind would be found to exist in all of the braces. Upon combining with the inclined reaction *ma* the vertical reaction also marked *ma* the direction of the supporting force will be found, and it is likely to be so much inclined to the vertical that provision against sliding on the wall plate at M should be made.

The stresses given by this diagram for wind on the left are found in the same table, in the column marked **W. L.** It will be seen that all of them agree in *kind* with those for steady load. It may perhaps not be amiss to suggest again how to determine the kind of stress on any member, without retracing the diagram from the beginning. Take any joint at which there is an external force. Its direction of action will be known. Pass over the line for that force in the proper direction, and then proceed round the polygon; the direction of movement over the respective lines denotes how each piece acts against the particular joint under consideration. If we move over a line from right to left for one joint, we must pass from left to right for its action on the joint at its other end.

TABLE OF STRESSES FOR FIG. 19.

PIECE.		Steady Load.	W. L.	W. R.
		lbs.	lbs.	lbs.
Rafters	BS	+8570	5600	8480
	CR	+6850	5600	6540
	EQ	+5700	5600	5880
	IP	+5700	5880	5600
	KO	+6350	6540	5600
	LN	+8570	8480	5600
Tie	LA	−7440	11400	0
	HA	−5450	7050	0
	DA	−5450	4850	2150
	BA	−7440	4850	6480
Braces	BC	+1720	0	3800
	CE	+1520	0	3300
	CD	−1000	0	2150
	FG	−2300	2500	2500
	GI	−1000	2150	0
	IK	+1520	3300	0
	KL	+1720	3800	0

The diagram for wind on the right is marked **W. R.** The supporting force at T, while still vertical, is greater in amount, and, if

diagram **W. L.** has been already constructed, can be taken as that portion of the vertical component of the wind pressure not included in $q\,a$ of that figure; that is $q\,a + t\,a =$ vertical component of wind pressure. If this should be the first diagram drawn, find the supporting forces in one of the three ways given above. The reaction at M is rightly denoted by $a\,p$, for, when the wind is on the right, there is no external force to divide the space from M to P.

The point a is moved considerably from its place in diagram **W. L.**, and this change affects the amounts of the stress in the horizontal member, but not in those pieces which bear similar relations to the two sides of the truss; in other words C B now has the stress of L K, etc. In some forms of truss, however, we find more material change. In the present example it happens that the vertical $f\,g$ strikes the point a, so that $i\,p$, the stress in the rafter, coincides with $a\,p$, the reaction at M; the wind on the right consequently causes no stress in L A and H A. The stresses from this diagram are found in the last column of the above table.

There is no occasion to tabulate the stress on K H, if that on I G is given, nor $g\,h$, if $k\,i$ is given. Notice that the joint K G or C F gives a parallelogram in each diagram, as alluded to before, the stress in I K passing to G H without change. It will be seen on inspection of the table that the combination of steady load with wind on the left gives maxima stresses in I P, K O, L N, L A, H A, D A, G I, I K and K L, while the remainder, with the exception of F G, have maxima stresses for wind on the right. F G is strained alike in both cases.

These wind diagrams might be drawn on either side of the line of wind force, as in the case of steady load, by changing the order in which the supporting forces are taken, or going round the truss and joint in the opposite direction. The different diagrams still resemble one another, especially if the comparison is made with one-half of the steady load diagram. Although there exists a four sided space C, the structure is sufficiently braced against distortion; for the space C is surrounded by triangles on all sides but one.

Turning to Fig. 20, we see that when the rafters do not slope directly from the ridge to the eaves, but are broken into two or more planes of descent, we shall have wind pressures of different directions and intensities on the two portions, I C and K B. After draw-

ing the steady load diagram, the method of finding the supporting forces which resist these two wind pressures will claim our attention. To make an example for practice we will assume that this truss, drawn to a scale of 40 feet to an inch, is 50 feet in span, height to ridge 20 feet, and to hips 14½ feet. The sides K B and G E are practically 16⅔ feet long, at an angle of 60° with the horizon, so that their horizontal projection is 8⅓ feet. The upper rafters are 17½ feet long, and therefore make an angle with the horizon of 18° 19′. The trusses are 8 feet apart, and are loaded at the joints only. The rafters would commonly be supported at intermediate points; but more lines would make our figures less plain.

The steady load is taken as 12 pounds per square foot of roof surface, or

$$(2 \times 16\tfrac{2}{3} + 2 \times 17\tfrac{1}{2})\ 12 \times 8 = 6560 \text{ pounds, total load.}$$

The joint L will carry one half the load on K B, or 800 pounds; $I K = \tfrac{1}{2} K B + \tfrac{1}{2} I C = 800 + 840 = 1640$ pounds; $I H = 840 + 840 = 1680$ pounds, etc. These weights are laid off in the diagram just above, on the right, for steady load, marked **S. L.** from *l* to *f* by a scale of 3,000 pounds to an inch. The diagram is now drawn without difficulty. The tensile stresses are marked —, and the compressive ones +. This diagram makes the rafters in compression and all the braces in tension.

In treating this truss as loaded with snow, it is considered that K B and E G are too steep for any weight to accumulate there, as whatever snow fell on them would soon slide off. Therefore a weight of 12 pounds per *horizontal* square foot, for the upper rafters only, is taken for the maximum snow load, and, as the horizontal projection of I C + D H is 33⅓ feet, that the load will be

$$33\tfrac{1}{3} \times 8 \times 12 = 3200 \text{ pounds,}$$

laid off from *k* to *g*, in the diagram marked **S**. The end portions, *k i* and *h g*, are each 800 pounds, and *i h* is 1600 pounds. The division into two equal reactions at the points of support gives *a*. This diagram much resembles the other, but there is one point worth noticing; the lines of stress, *i c* and *h d*, cross in the first diagram, but do not in the second; while the reverse is the case with *e d* and *b c*. The result is that the stress of C D is reversed by the maximum snow load, and, as this stress is greater in amount than the one for the weight of roof, etc., C D will be a compression member when-

ever such a load of snow falls on the roof; and will be in tension when the snow is removed. The stresses from these two diagrams are given in the table on page 43, in the columns indicated, and the kind of stress is shown by the accompanying sign.

If the wind blows horizontally from the left, with a force of 40 pounds on a vertical square foot, and the truss has no rollers under either end, the diagram at the top, marked **W. L.** 1, will be obtained, the wind being considered by itself. The polygon of external forces must first be constructed. From the table of wind pressures given on page 32, we see that the intensity of pressure on K B will be 40 pounds, and on I C will be 16.9 pounds, normally, per square foot of roof. The total pressure on K B will therefore be

$$40 \times 16\tfrac{2}{3} \times 8 = 5{,}334 \text{ pounds,}$$

of which one-half will be supported at the joint L, and the other half at the joint J, as indicated by the two arrows perpendicular to K B. The pressure on I C will be

$$16.9 \times 17\tfrac{1}{2} \times 8 = 2{,}366 \text{ pounds,}$$

or 1183 pounds at each joint.

In the diagram marked **W. L.** 1, then, $hi = 1183$ pounds, on a scale of 3,000 pounds to an inch: $ij = 1183$ pounds, and $jk = 2667$ pounds, the two components at J; and $kl = 2667$ pounds, the wind pressure at L. For ij and jk may be substituted ik, if desired, the resultant of these two components.

To find the supporting forces:—Prolong the resultants of the wind pressure from the middle point of each rafter to intersect the line L F. The resultant K will be resisted at L and F by two reactions parallel to it, and proportional to the two segments into which this resultant divides L F, as was shown for Fig. 18. The same will be true of the resultant I. By scale, or from the known angles, it will be found that the resultant K cuts L F at 16⅔ feet, or one-third the span, from L, and the resultant I cuts it at 22.4 feet from the same end. Dividing jl at ⅓ of its length, we have la' for the reaction at L, and $a'j$ for the reaction at F. If we divide hj at $\frac{22.4}{50}$ of its length, ja'' will be the supporting force at L, and $a''h$ at F. By drawing the parallelogram $a'ja''a$ we shall bring the reactions for each wall together, and shall have, for the supporting force at L, or A L, la' and $a'a$, or their resultant la; and for that at F, aa'' and

a'' h, which combined give *a h*, properly called A H in the truss below, since the letters from F to H are not in use at present.

The polygon of external forces, when there is no roller under the truss, is therefore *h i*, *i k*, *k l*, *l a*, and *a h*. The completion of the diagram, by drawing lines parallel to the several pieces, will be easy without further explanation. The stresses obtained can be compared with those for steady load, and added algebraically to them for the combined action. That the point *e* should apparently fall on *i k* is only accidental. The signs affixed to the lines will enable one to see readily that the stresses in B C and E A are now reversed, the pressure I K obliging us to use a strut to keep that joint in place. The resultant, however, from the combined stresses on E A is still tensile. The amounts given by diagram W. L. 1 have not been tabulated, as we preferred to treat this truss from another point of view. Had they been placed in the table, it would be unnecessary to draw a diagram for wind on the right, for the different members of the truss would exchange stresses symmetrically; that is, A B would have the stress of E A, and E A that of A B; D H of C I, etc., C D remaining the same. The rest of the table could therefore be completed by inspection.

If rollers are placed at L, to allow for movement resulting from change of temperature, the supporting forces will be modified, L A becoming vertical. The diagram marked W. L. 2, shows the effect of this change. So far as drawing the lines of wind pressure *h i j k l*, the polygon of external forces will be obtained in the same manner as before. We may then draw the parallelogram and locate the point here marked *a'*; then draw *a' a* horizontally, and we shall get *l a*, the vertical reaction at L, equal to the vertical component of *l a* of the figure just preceding.

In case the former diagram has not been drawn, a readier way to determine *l a* will be as follows: — Draw *h l*, plainly the resultant of *h j* and *j l*; then, having prolonged the dotted arrows at I and K until they meet, draw a line, parallel to *h l*, through their intersection. This line will give the position of the resultant of the wind pressures, and *l h* is now to be divided in the ratio of the two segments into which the resultant divides the span L F. The point of division will fall at *a''*, where *l h* is crossed by *a' a*, and the point *a* can thus be located without the parallelogram. The work is less,

as there will be but one point to be determined instead of two. This method will not answer for finding the supporting forces if they are both inclined, as it would give L A and A H parallel to one another. The reaction at L being *l a*, the one at F is *a h*, requiring the resistance at F of the entire horizontal component of the wind pressure.

A comparison of the two **W. L.** diagrams will show that the stress in every piece is changed very decidedly in amount, and that in a number of the pieces the stresses are reversed. Those obtained wtth the rollers at L are tabulated with their proper signs in the column marked **W. L.**

TABLE OF STRESSES FOR FIG. 20.

PIECE.		S. L.	S.	W. L.	W. R.	−	+
Braces	A B..	−1,790	−1,160	+1,650	−1,300	4,250	
	B C..	−1,050	−1,040	+3,250	−2,560	4,650	2,200
	C D..	− 90	+ 250	+ 750	−1,160	1,250	910
	D E..	−1,050	−1,040	− 500	+1,300	2,590	250
	E A..	−1,790	−1,160	+5,250	−4,970	7,920	3,460
Rafters	E G..	+3,420	+2,200	+ 550	+3,470		9,090
	D H..	+2,800	+2,200	+ 800	+3,470		8,470
	C I...	+2,800	+2,200	+ 420	+3,800		8,800
	B K..	+3,420	+2,200	+1,450	+2,600		8,220

When the wind blows from the right, the diagram marked **W. R.** will be drawn. The lines *i h g f*, representing the wind, will correspond in value with *h i k l* of the preceding figure, and, since the other diagram was constructed first, the vertical reaction at L will now be obtained by drawing the horizontal line *a' a*, from either the angle of the parallelogram or the proper point of division of the resultant *i f*, so as to give *a i*, the smaller part of the vertical component of the wind pressure; that is, *l a*, from **W. L.** 2, plus *a i*, from **W. R.** equals the vertical projection of the polygon of external forces.

When this diagram is completed by the customary rules, a comparison of it with the one just preceding will make clear the effect of wind on different sides. The stress in the rafters is much greater when the wind blows on the side farthest from the rollers, but it is always compressive. The forces in the braces are all reversed.

The weight of the roof and truss may be the only external force, or snow may be added; and, in either case, the wind may also blow

on one side or the other. Selecting then from the table those stresses which may exist together, we find the maximum tension and compression in each piece, as given in the last two columns. The rafters are always compressed, and A B is always in tension. The other pieces must be designed to resist both kinds of stress, although the compression on D E is quite insignificant.

If a truss has a curved outline, the pressure of the wind will make a different angle with the horizon for every point. But there will be no sensible error if the pressure on each piece is assumed to be normal to its curve at its middle point, or, what is sensibly the same thing, perpendicular to the straight line joining its two extremities. Thus in the truss of Fig. 21, the wind pressure on C T is taken as perpendicular to a straight line from B to the next joint in the rafter.

We will first give the dimensions used for this truss, which is here drawn to a scale of 30 feet to an inch. The span is sixty feet; height at middle of rafters 15 feet, at middle of main tie 6 feet. The curves are arcs of circles, the radii of the upper and lower ones being respectively 37½ feet and 78 feet. The rafters are spaced off at intervals of 11½ feet each way from the middle, and the tie is divided into 10¼ feet lengths. The end pieces will differ slightly from these measures. The trusses are 10 feet apart. From the data, radius 37½ feet, and half chord or sine 5¾ feet, it is easy to calculate that the chord of the first piece of rafter from the middle will make an angle with the horizon of $8^\circ\ 49\frac{1}{4}'$. The second piece will be inclined three times as much, or $26^\circ\ 28'$, and the last, five times as much, or $44^\circ\ 6'$. The intensity of normal wind pressure will then be, when interpolated in the table of page 32, 8.6 lbs. per square foot for the upper length, 23.7 lbs. for the next length, and 35.6 lbs. for the lowest piece. Multiplying these intensities by $11\frac{1}{2} \times 10$, we get 989 lbs., 2725½ lbs. and 4094 lbs. respectively, represented by the small arrows, as if concentrated at the middle points of E, D and C. The steady load is taken at a small figure, 2300 lbs. per piece of rafter, to allow the disturbing effect of the wind to be more marked.

It is proposed that the diagonals for this truss shall be light iron rods, not adapted to resist compression, and therefore, if a compressive stress would occur in a particular diagonal, in case it were alone in a panel, we substitute the other diagonal, which will then be in

tension. This statement was explained in connection with the trapezoidal truss, Fig. 13. In lettering the figure, that *tie* which is required for a particular distribution of load is supposed to exist, and the other diagonal is not taken account of. Thus, considering the panel through which the dotted arrow is drawn, if the brace which goes from the top of O P to the bottom of Q R is under stress, it will be called P Q, while the rafter will be Q E and the bottom tie P A. If the other diagonal is strained, the rafter will be called P E and the main tie Q A.

The diagram for weight of roof and truss is just above Fig. 24, and is drawn by laying off the loads from *i* to *b* and then taking up the joints in succession. The scale is 8000 lbs. to an inch. The point of support B gives *c b a t c*. On taking the next joint in the top or bottom member, we find three pieces whose stresses are unknown. Both diagonals R S cannot be in action as ties at once: therefore suppress one, for instance that which runs to the upper end of S T. We shall then have only two unknown stresses at the upper joint, and can draw ts' and $s'd$. The lower joint will then give $s't$, ta, ar', $r's'$. But $r's'$ will be a compressive stress, as we just moved from r' to s'. This diagonal is therefore not the right one. Taking the other, and trying the lower joint first, we have *t a s t*, and the upper joint then gives *d c t s r d*. Analogy will lead us, and rightly, to take the other diagonals which slope the same way, as far as the middle, and then, because of symmetry of load, we take those which incline the other way. It is therefore easy, after the first attempt, to decide which diagonal to reject and which to retain.

It will be seen how small the stresses are throughout the bracing, compared with those in the main members. If *d r* had been slightly more inclined, so as to strike *s*, no diagonal R S would have been required for this distribution of load. All of the pieces of the lower member, being denoted on one side by A, will have their stress lines radiating from *a*. The length of *h k*, etc., as compared with that of H K, &c., shows the necessity of drawing the truss skeleton on a large scale, so as to secure parallelism of the respective lines in each figure.

We may analyse the effect of the wind separately upon the truss, but, as there is a likelihood that the wind will reverse the stress on

some of the diagonals which experience tension from the steady load, and that we shall be obliged, therefore, to substitute the other diagonals in such panels, it seems better to draw the diagram for the wind and the weight of the roof in conjunction. After it is constructed, by comparison with the diagram for vertical load, just drawn, what portion is due to wind alone can doubtless be ascertained. The two diagrams above the truss, marked **W. R.** and **W. L.**, are drawn for the maximum force of wind on either side, combined with the weight of the roof, etc.

It will be seen that the external load line *b i* of one case is the exact reverse of *i b* of the other. An explanation of the construction of **W. R.** will suffice for both. As the wind blows from the right, there is only the steady load on the left half of the truss. Beginning therefore with the joint at I, we will lay off vertically *h i*, equal to one-half the load on H K, or 1150 lbs. Next $g h = 2300$ lbs., load at G H, and so on, corresponding exactly with the load line of the steady load diagram, discussed before, as far as the joint F E. Here we find, in addition to 2300 lbs. vertical pressure, an inclined force perpendicular to the tangent at E, or to the chord of the piece, and equal to one-half of 989 lbs., the wind pressure before computed for E. This gives the inclined line as far as *e* in the diagram. The next joint gives *d e*, manifestly made up of the other half of 989 lbs., of the vertical 2300 lbs., as usual, and finally of one-half of 2725½ lbs. from the next length of rafter, and perpendicular to it. The forces for the remaining joints C D and B C will be plotted in the same manner, and we therefore see that, commencing at B, as is proper for this load line, we lay off the vertical and inclined forces in regular succession from one side of the truss to the other. If one desires to draw a straight line from *c* to *d* it will be the resultant of the combined forces at C D.

Connect *b* with *i* by the dotted line. This line will be the resultant of all these forces from *b* to *i*. As the resultant of the dead weight which is symmetrically distributed acts in the line of the vertical O P, and hence through the centre of curvature of the rafters, and as the wind pressures all point to the same centre of the circle, the resultant *b i* must pass through the same point. Therefore through that point draw the dotted arrow parallel to *b i*. This arrow cuts the span B I, by measurement, at 25.24 feet from B, or

34.76 feet from I. The resultant $b\,i$ scales 20,620 lbs. If the supporting force at B were parallel to this resultant, it would be found by taking moments about I, when we should have

$$B \times 60 = 20{,}620 \times 34.76 \quad \text{or} \quad B = 11{,}946 \text{ lbs.}$$

Lay off this force from b to a'. If rollers are placed at B, that reaction will be vertical, and the horizontal component of $a'b$ must be resisted at I. Let fall $b\,a$ vertically, determining the point a by drawing $a'a$ horizontally, and connect a with i. The two supporting forces will be $i\,a$ and $a\,b$.

If there are no rollers under the truss, pay especial heed to what was said in regard to Fig. 19, and find the supporting forces for each oblique pressure separately. This same thing must be done when the curve of the rafters is not circular, as the forces will not then meet at a common centre. In the **W. L.** diagram the point a' will come nearer to b than to i, — that is the quantity just obtained applies to the point of support I, — and a falls very near to, but just outside of f, in the prolongation of the vertical line. Having thus completed, in either case, the polygon of external forces, the remainder of the construction will be made as in any example. After the first trial to ascertain the proper diagonal, it appears that, in each case, the diagonals all slant one way; so that, for wind on one side, one set of diagonals is in tension, and, for wind on the other, all of the other set are strained.

The effect on the five pieces of a panel, top, bottom, two sides and the diagonal, of drawing the diagram so as to give compression on a diagonal, is shown in the **W. L.** figure for the panel P Q. Instead of $o\,p$ and $q\,r$, we get $o\,p'$ and $q'\,r$, considerably increased in amount but the same in kind; for $e\,p$ and $a\,q$ are substituted $e\,q'$ and $a\,p'$, unchanged in kind, but practically what is taken in amount from one is added to the other; while the diagonal stress is, as we said, reversed, but very nearly the same in amount.

It might be practicable to deduce some rule for determining beforehand the diagonal parallel to which one should draw a line, but the tentative process seems easy. We find it convenient to draw the lines parallel to the rafter and main tie, as $e\,p$ and $a\,p'$, first, then to sketch roughly two lines for the suspending piece and diagonal, see whether that diagonal comes in tension, and finally draw the right ones carefully. It is not necessary to put signs of + or − on

these lines, for it may be seen that all the parts of the rafter are compressed, the whole lower member extended, and all the diagonals in tension, as well as all the suspending pieces except O P and Q R, which are compressed to a very small amount when the maximum wind comes from the right. Such pieces are easily selected, if one notices that *op* and *qr* in the **W. R.** diagram are drawn in a direction opposite to the common one.

The stresses are given in the following table. The lengths of rafter are denoted in the table by a single letter. The pieces of the main tie, having the letter A in common, have also the letters which

TABLE OF STRESSES FOR FIG. 21.

		S. L.	W. R.	W. L.	+
Rafters .	C....	12,600	18,900	16,200	18,900
	D....	11,400	17,500	15,600	17,500
	E....	10,800	15,000	16,200	16,200
	F....	10,800	13,300	17,900	17,900
	G....	11,400	12,700	20,100	20,100
	H....	12,600	13,100	21,800	21,800
					—
Main Tie.. A	..	K 9.600	K 5,500	K 19,500	19,500
	..	L 9,500	L 5,500	M 18,000	18,000
	..	N 10,400	N 7,200	O 16,000	16,000
	..	Q 10,400	P 9,000	Q 14,200	14,200
	..	S 9,500	R 10,900	S 12,300	12,300
	..	T 9,600	T 12,800	T 12,300	12,800
Diagonals.	L M	╲ 900	╲ 1,800	╱ 1,800	
	N O	" 400	" 2,100	" 2,400	
	P Q	╱ 400	" 2,400	" 2,200	
	R S	" 900	" 2,200	" 2,100	
					—
Suspenders.	K L	1,200	700	1,200	1,200
	M N	1,000	200	900	1,000
	O P	900	+ 100	700	900
	Q R	1,000	+ 50	1,000	1,000
	S T	1,200	400	1,600	1,600

stand before the stresses in the proper columns. The inclination of the diagonal is shown by the sign prefixed to the stress. The effect of the wind on the right, or on the same side with the rollers, is to materially reduce the stress on a large portion of the main tie. The light bracing required is a marked feature of this type of truss, due to its resemblance to a curve of equilibrium for a load concentrated at a series of points. The predominance of tensile members makes it well adapted for construction in wrought iron. If the diagonals

are all made of the same section, to safely withstand a tensile stress of about 2400 lbs., they will be as nearly adapted to the case as is practicable, and the suspending pieces may be proportioned to a stress of 1600 lbs. The two compressions, marked +, are too insignificant to require an increase of section.

If a fall of snow is supposed to be uniformly distributed over the roof, the increased action of the several pieces of the truss can be readily obtained by proportion from column S. L. But, if it is thought that the inclination of the portions near C and H is too great to admit of this approximation, a new diagram for snow should be drawn. The horizontal projection of a piece of the rafter is properly taken when reckoning a snow load.

We think the reader will have no difficulty in drawing diagrams for a truss of similar outline, but with only one system of triangular bracing, something like the Warren girder of Fig. 15.

CHAPTER VII.

BENDING MOMENT AND MOMENT OF RESISTANCE.

Having treated of the action of external forces upon a great variety of trusses, we propose now to investigate the graphical determination of the bending moments which occur on certain pieces.

To recapitulate some statements of an earlier chapter: —

In case the transverse components of the load upon a portion of a rafter, or other piece of a truss, are not immediately resisted by the supporting power of some adjacent parts, or, in other words, unless the load on the structure is actually concentrated at the several joints, such transverse components will exert a bending action on the portion in question, and the additional stress thus caused in the piece may be too great to be safely neglected. Further, in case the piece makes any other than a right angle with the line of action of the load, or has an oblique force acting upon it, the stress along it, given by the diagram, will be less than the maximum, and will generally be the mean stress. Lastly, in case a piece is curved, a bending moment will be exerted upon it by the force acting along the straight line joining its two ends, this bending moment being a maximum at the point where the axis or centre line of the piece is farthest removed from the line drawn between its ends.

The reader is referred for an illustration of the truth of some of the statements to page 29, Fig. 17; but, to take another and a simple example:

Suppose the rafters A C and B C, Fig. 22, to be loaded uniformly over their whole extent. Let us assume, in the first place, that the tie A B is not used, but that the thrust of the rafters is resisted by the walls which carry the roof. Consider the piece A C. Since the roof is symmetrically loaded, the thrust at C must be horizontal, and therefore the reaction which supports this end of A C will lie in

the line C E. The centre of gravity of the load on A C being at D, its middle point, the resultant of the load will, if prolonged upwards, intersect C E at E. Since the rafter is in equilibrium under the load and the reactions at C and A, the direction of the reaction of the abutment or wall at A must also pass through E. (Compare Figs. 3 and 4.) Draw A E and prolong E D to G. Let E G be measured by such a scale as to represent the load on A C. The three forces meeting in the common point E will then be equal to the respective sides of the triangle A E G, drawn parallel to them; and, since A G equals E C, the reactions at A and C will be A E and C E.

Decompose A E and C E into components along and transverse to the rafter, and we shall have A F = direct compression on the rafter at A, while C F = direct compression at C. The compression on successive sections of the rafter increases from C to A by the successive longitudinal components of the load. The two components A L and C Q, which, combined with A F and C F, give the original forces A E and C E, are analogous to the supporting forces of a beam or truss, and from them we obtain the amount of the bending action of the load on this rafter. If, now, the rafters simply rest on the wall, being secured against spreading by the tie A B, the reaction A E will be replaced by the two components, A I, the upward supporting force of the wall, and A G, the stress exerted on the tie; but these two forces give the same stress on the rafter as before.

Consider, next, the method by diagram. The load is now to be concentrated at the joints, and we shall have, in place of E G, A N and C P, each one-half of the load on one rafter. Lay off 1-2 to represent the total load on the roof, make 1-3 = A N, 1-4 = A I, and draw 3-5 and 4-5 parallel to the rafter and tie. A G will equal 4-5, and therefore the stress on the tie is given correctly; but, since A I − A N = A K = 3-4, 3-5 equals A D, and this is the stress given by the diagram as existing from A to C, a supposition which is true when the load is actually concentrated at the joints, but is not true for a distributed load. But A D, or 3-5, is equal to one-half of A F + F C, and is manifestly the value of the direct compression at the point D of the rafter; all of the load from A to D was, when we drew the diagram, considered to be concentrated at the joint A.

The load on the principal rafters of a roof-truss is generally concentrated at a series of equidistant points, by means of the *purlins*, or short cross-beams which extend from one truss to another, and which are themselves weighted at a series of points by the pressure of the secondary rafters. These secondary rafters, when employed, carry the boards, etc., and thus have a uniformly distributed load. It is only in cases where purlins rest at other points than the *so-called* joints that bending action occurs in the principal rafters, or in very light trusses where the boards are nailed directly to the main rafters.

It will first be well, even at the risk of dwelling on what is well known to many, to explain what *bending moment* and *moment of resistance* are. A beam A B, Fig. 23, supported at its two ends, when loaded with a series of weights, distributed in any manner, is in equilibrium under the action of vertical forces, the weights acting downwards and the two supporting forces acting upwards. The supporting forces are easily calculated by the principle of the lever, or by taking moments as explained for Figs. 13 and 16. As the beam is at rest, there must be no tendency to rotate, and therefore, if we assume any point for an axis, the sum of the moments, that is of the products of each force by its distance from the axis, must equal zero. A moment which tends to produce rotation in one direction being called plus, one which acts in the other direction is called minus. If then we pass an imaginary vertical plane of section through any point in the beam, such as E, the sum of the moments on one side of the plane of section must balance or equal that on the other. The sum of these moments on one side or the other is called the *bending moment:* why will be seen soon.

These moments can balance one another only through the resistance of the beam at the section in question, or of the stresses, exerted between the particles cut by this section, which resist the tendency to bend the beam. The beam becomes slightly convex, and the particles on the convex side are extended, while those on the concave side are compressed. Experiment shows that, for flexure within such moderate limits as occur in practice, the horizontal forces exerted between contiguous particles vary uniformly as we go from the top of the beam to the bottom, the compressive stress being most intense on the concave side, diminishing regularly to zero at

some point or horizontal plane, called the neutral axis, then changing to tension and increasing as we approach the convex side. The two sets of stresses reacting against each other may be represented to the eye by the arrows in the vertical section marked E′.

Since all of the external forces are vertical, these internal stresses, being horizontal, must balance in themselves, or the total tension must equal the total compression, whence it follows that the neutral axis must pass through the centre of gravity of the section. To make this clear, let one consider how the centre of gravity or the position of the resultant of parallel forces is found. (Compare Fig. 16). We multiply each force or weight by its distance from any assumed axis, and divide by the combined weights. Now if we attempt to find the centre of gravity of a thin cross-section of this beam, and take our axis at the point where the centre of gravity happens to lie, we can see that the distance of each particle from the axis will vary exactly as these given stresses; hence the neutral axis must lie in the centre of gravity of each cross section. As these stresses are caused by the resistance to the bending moment on each side of the section, the moment in the interior of the beam, made up of the sum of the stress on each particle multiplied by its distance from the neutral axis, or indeed from any axis, and known as the *moment of resistance*, must equal the bending moment at the given section. As the tensions and compressions on one side of the plane of section tend to produce rotation about the neutral axis in the same direction, their moments are added together.

The bending moment, then, in the beam A B of the figure, at any section E, will be, if P is the supporting force on the right, W, W′, &c., the weights,

$$P.BE - W.CE - W'.DE;$$

or, in general, if L equal the arm of any weight, and Σ be the sign of summation,

$$M\,(\text{the bending moment}) = P.BE - \Sigma\, W.L,$$

it being remembered always to take only the weights between one end and the plane of section.

If the reader will take a special case, and, having a beam of known length with weights in given positions, will first find the supporting forces, and then calculate the bending moment on either side of a plane of section, he will obtain the same result with oppo-

site signs, showing that the two moments balance one another. The numerical result, being the product of two quantities, is read as so many foot-pounds or inch-pounds, according to the units employed. As the stress on any material is usually expressed in pounds on the square inch, the latter units are the better. The moment of resistance being numerically equal to the bending moment, and therefore equal to the above expression, it is practicable, for a given beam and load, to determine the maximum stress at any section, or, knowing the proper working stress to which to subject the material, to determine the required cross-section.

The weights on one side of the section may all be considered to be concentrated at their common centre of gravity, or point of application of their resultant, so far as the bending moment at that section is concerned, and the load when continuous is always so taken.

Let us suppose that the weights which, in Fig. 23, rest upon the beam are suspended by a cord from the points *a* and *b*, at the same level, being attached to the cord at the successive points *c*, *d*, *f* and *g*, vertically below their former positions at C, D, F and G. Let us further suppose that the amount of the weight at G alone is at present known. This cord can be treated as if it were a frame. Taking the joint *g* into consideration, draw 5-4, vertically, equal to the weight, then 5-0 parallel to *a g*, and 4-0 parallel to *g f*. The two lines just drawn must be the tensions in *a g* and *g f*. For the joint *f*, *f g* is now known; therefore 4-3 parallel to the weight, and 3-0 parallel to *f e* will determine the other forces at *f*. The side 4-3 must equal the weight at F, and must lie in the same straight line with 5-4; for this triangle was constructed on the side 4-0 previously found. Continuing the construction for the successive angles of the cord, we find that a vertical line 5-1 will represent by its several portions the successive weights, and that the tensions on the different parts of the cord will be given by the lines parallel to these parts, drawn from the points of division of the load line, and all converging to the common point o. Draw o-6 horizontally, parallel to *a b*; this line will be the horizontal component of the tension at any point of the cord, and is here denoted by H. The form assumed by the cord for a given distribution of weights may be called the *curve of equilibrium*, as the system will be in equilibrium or at

rest; and it is also called in mechanics a funicular polygon. Students of mechanics will recall the fact, so easily shown here, that the horizontal component H is a constant quantity at every point.

If now the cord, instead of being fastened at *a* and *b*, is attached to the two ends of a rigid bar *a b*, and the whole system is then suspended from A and B by two short cords, its equilibrium will not be disturbed. The pull 5-0 at *a* will be decomposed into 0-6, compression on *a b*, and 6-5, tension along *a* A. Similarly at *b*, 0-1 will be decomposed into 1-6 along *b* B and 6-0 along *b a*. 6-0 balances 0-6, while 1-6 and 6-5 must be the supporting forces at *b* and *a*. As the supporting forces do not depend upon the form of the frame or truss, the reactions which carry the beam at B and A must be these same quantities.

We may make the construction more general by drawing a ***curve of equilibrium*** from any point *a'*, vertically below A, that is, by determining the outline of a cord which will sustain in equilibrium the given weights at the given horizontal distances from A. Lay off the weights in succession from 5 to 1; assume any point 0' arbitrarily and connect it with all the points of division of the load line. Commence at *a'*, and draw *a'g'* parallel to 5-0', stopping at the vertical dropped from G; then draw *g'f'*, parallel to 4-0', etc., and finally *c'b'*, parallel to 1-0'. That this will be the figure of a cord suspended from *a'* and *b'* follows from the preceding demonstration. Connect *b'* with *a'*, and the line, parallel to *b'a'*, from 0' must strike the same point 6 which the line from 0, parallel to *b a*, touched. The supporting forces, if *b'a'* exists, will be 1-6 and 6-5 as before; but 0'-6' will be the horizontal component H' for this cord.

If we turn again to the first cord, attached at *a* and *b*, the piece *a b* being dispensed with, and take any point of it *e*, the moment of all the forces on one side of the point must be the bending moment there; but as the cord is perfectly flexible and is at rest, this bending moment will equal zero. Using, instead of 1-0, its two components 1-6 = P and 6-0 = H, multiplying each force by the perpendicular distance of its line of action from *e*, calling the combined moments of the weight on one side of *e* $\Sigma W.L$ as before, and denoting the tendency to produce rotation in opposite ways by opposite

signs, we shall have, for moments of forces on the right of, and around e,

$$P \, . \, bk - \Sigma W \, . \, L - H \, . \, ek = 0,$$

or

$$H \, . \, ek = P \, . \, bk - \Sigma W \, . \, L.$$

But $P \, . \, bk = P \, . \, BE$, and $P \, . \, BE - \Sigma W \, . \, L = M$, the bending moment at the section E of the beam, as shown on page 53, therefore

$$M = H \, . \, ek.$$

By a similar analysis of the lower cord we have

$$P \, . \, ik' - \Sigma W \, . \, L = (6\text{-}0') \, . \, e'l = M.$$

From similarity of triangles $le'k'$ and $6'0'6$, we have

$$e'l : e'k' = 6'\text{-}0' : 6\text{-}0',$$

or

$$(6\text{-}0') \, . \, e'l = (6'\text{-}0') \, . \, e'k';$$

therefore

$$M = (6'\text{-}0') \, . \, e'k' = H' \, . \, e'k'$$

as in the other case. The treatment is therefore general, and the bending moment at any section of the beam equals the product of H from the stress diagram by the vertical ordinate, below the section, from the cord to the line connecting its two extremities.

The relative situations of a' and b' will depend on the choice of the position of $0'$, and this point may be taken wherever convenient. H' is measured by the same scale used for plotting 5-1, while $e'k'$ must be measured by the scale to which A B is laid off. The two scales, one representing pounds and the other inches, need not be numerically the same. Their product will be inch-pounds. A single load on the beam will have for its curve of equilibrium two straight lines from a' and b', meeting at a point vertically under the weight. A uniformly distributed load will give a parabola with the maximum ordinate at the middle of the span. This load may be treated as if concentrated at any convenient number of points along the beam, as we have done in getting the loads at the several divisions of a rafter, and the angles of the polygon will lie in the desired parabola. When the beam is inclined the transverse components only of the load produce any bending, as explained for a uniform load in Fig. 22.

To determine next the moment of resistance for a particular form of cross-section: —

Let us consider first a beam of rectangular cross-section, represented by A B C D of Fig. 24. The intensity of stress, as shown at

E′, Fig. 23, varies uniformly each way from the neutral axis. The neutral axis, lying through the centre of gravity, G, of the cross-section, will be at E F, the middle of the depth. The stress on the square inch will be most intense on the fibres at the edge A B or C D, and less intense on any intermediate layer, such as I K, in the proportion of E I to E A. If then we draw from G the lines G A and G B, and imagine that the layer I K is replaced by I′K′, which has its breadth diminished in the same proportion, the total stress on I′K′, if of the intensity found at A B, will be equal to the total stress of less intensity actually existing on I K. This stress will also have the same leverage about E F with that on I K. By the same reasoning for all layers of the beam, we obtain the two triangular, shaded areas, A B G and G D C, one of which, usually the upper one, when multiplied by the maximum intensity of stress, represents the total compression, and the other the total tension at the section. The moment of either force about the neutral axis will be most readily obtained by considering the stress, which is now uniformly distributed over the triangle, as concentrated at its centre of action, the centre of gravity G′ of the triangle. This centre of gravity is at two-thirds of its height from the apex G.

Letting b represent the breadth and h the height of the cross-section in inches, the area of one triangle will be $\frac{1}{2}b \cdot \frac{1}{2}h$; the lever arm will be $\frac{2}{3} \cdot \frac{1}{2}h$; and if f represent the maximum stress on the square inch at A B, since the tension and compression tend to produce rotation in the same direction, we add the moments of the two forces together and have

$$2\left\{\frac{bh}{4} \cdot f \cdot \frac{1}{3}h\right\} = \text{moment of resistance} = \frac{1}{6}fbh^2.$$

Putting this equal to the bending moment M, we obtain

$$H'\,e'k' = \frac{1}{6}fbh^2.$$

If we select the maximum value of $e'k'$, introduce the safe working stress f for the extreme fibres, and assume either b or h, we can compute the other dimension. This will determine the beam when of uniform section throughout. If the cross-section is to vary, it will be proportioned to the bending moment at different points. As the stiffness of the beam depends principally upon h, the depth must not be made too small.

It is easier to compute the size of a beam of rectangular cross-section by the above formula than to draw Fig. 24, but, for a less regu-

lar section, where the areas and centres of gravity of the shaded portions are not so readily obtained, the graphical method for finding the moment of resistance is convenient.

In applying this method to a beam of the section shown in Fig. 25, we must first ascertain the centre of gravity of the section. By multiplying each rectangular area by the distance of its centre of gravity from either the top or bottom, adding the products, and dividing by the whole area, we find the distance of the neutral axis N O from that edge. If G I $= b$, A B $= b'$, G E $= h$, and C A $=$ h', we have

$$\frac{b h . \frac{1}{2} h + b' h' (h + \frac{1}{2} h')}{b h + b' h'} = \text{distance of neutral axis from G I.}$$

The construction of the shaded area A P B needs no explanation, as the remarks on Fig. 24 apply equally well here. The stress on the fibres at the edge G I will not be so great as at the edge A B, because they are not so far from the neutral axis. If the fibres at G I were removed to K L, so as to be equally remote with A B, they would be equally strained. Then to reduce the layer G I to one which, if it had the same intensity of stress with A B, would give the same total stress which now exists on G I, project G I to K L, draw K P and L P, and G′ I′ will be the desired reduced length. The remainder of the shaded area for the lower rectangle follows the usual rule. In the same way the fibres at C D will be projected at Q R, and, by drawing Q P and R P, we determine C′D′, and thus complete the shaded portion. These triangles, etc., can be readily scaled, or computed from known proportions of the beam, their centres of gravity found and the moment of resistance calculated.

This graphical method for finding the moment of resistance of a given cross-section was published in the English paper "*Engineering*" several years ago, and also in Baker's book "On the Strength of Beams, Columns and Arches" It can also be found in Wood's "Resistance of Materials," and Van Nostrand's Science Series, No. 19.

A good example of a beam whose moment of resistance is not readily determined by calculation is afforded by a deck-beam, often employed in floors and roofs, and shown in Fig. 26. This example is taken from the Phœnix Iron Co.'s book of shapes, and is drawn

to one-quarter scale. The height of the section is 6 inches, breadth of flange A B $3\frac{1}{2}$ inches, thickness of web $\frac{3}{8}$ inch, weight per yard about 44 lbs.; therefore the area of cross-section is about 4.4 square inches.

The readiest way to determine the moment of resistance of this cross-section is as follows:—

Trace its full size from the book of shapes and transfer the outline to some heavy paper by going over the lines with a blunt point; cut the section from the heavy paper and determine its neutral axis by balancing it over a knife edge. Thus is found the line C D. Draw K L horizontally at the same distance from C D that S T is. A B will be projected at K L, and lines from K and L to P, the middle point of C D, or the centre of gravity of this section, will cut A B at A′ and B′, making A′B′ the reduced length of A B, which may now be consiaered to have the same stress per square inch as exists at I G. In the same way the end M of M N will be projected at O, the point U at V, and the lines from O and V to P will cut the horizontal lines through M and U at new points in the desired curve. Thus enough points are determined to locate the boundary of the shaded portion from B′ to P. The portion of the web with parallel sides gives of course a triangle, found at once by drawing a line from W to P. The curve A′P corresponds with B′P. For the upper portion, project E F on T S, draw lines to P, and get in a similar way enough points for this curve. Cut out the two shaded figures, balance each one over a knife edge and thus determine their respective centres of gravity Q and R. Calculate the area of one; the area of the other should exactly equal it, for the total tension equals the total compression. Calling this area A and the safe working stress on the square inch f, we shall then have for the moment of resistance

$$f.A.PQ + f.A.PR = f.A.QR.$$

In this example A = 1.29 sq. inches, P Q = 2.12 inches, and P R = 2.66 inches. If therefore f = 15,000 lbs., the moment of resistance equals

$$15{,}000 \times 1.29 \times 4.78 = 92{,}493 \text{ inch pounds.}$$

In simpler cases, the required size of beam to sustain a given load is more readily found by formula. If I beams are used, the web being thin, and the top and bottom flanges alike, an approximate

formula may be used. If F represents the area in square inches of the cross-section of either flange, W the area of the web, h the depth from centre to centre of the flanges, and f the safe stress on the square inch, the moment of resistance is nearly equal to

$$f h \left(F + \frac{W}{6}\right).$$

The value of the breaking stress under a transverse load may be taken, for wood, at 9,000 to 10,000 lbs.; for wrought iron, 40,000 to 45,000 lbs. These quantities must be divided by the factor of safety, explained presently, to give f of these formulæ.

CHAPTER VIII.

LOAD AND DETAILS.

The principal trusses should be braced together in the planes of the rafters to prevent wind, in a direction perpendicular to the gable ends, from producing any lateral movement. It is often customary also to tie the trusses down to the walls, especially in those buildings, partially open, where wind may get below the roof.

It has been a very common practice to assume the steady load as 40 lbs. per square foot of roof, including truss. The various items may be computed separately, and an additional amount assumed for the frame. After the truss is roughly designed its weight should be calculated to see how well it agrees with the assumed weight. If the agreement is not sufficiently exact, the proper allowance is then to be made.

Trautwine says that, for spans not exceeding about 75 feet, and trusses 7 feet apart, the total load per square foot, including the truss itself, purlins, etc., complete, may be taken as follows:—

Roof covered with corrugated iron, unboarded	8 lbs.
Same if plastered below the rafters	18 lbs.
Roof covered with corrugated iron, on boards	11 lbs.
Same if plastered below the rafters	21 lbs.
Roof covered with slate, unboarded or on laths	13 lbs.
Same on boards 1¼ inches thick	16 lbs.
Same if plastered below the rafters	26 lbs.
Roof covered with shingles on laths	10 lbs.

For spans from 75 feet to 150 feet it will suffice to add 4 lbs. to each of these totals.

The weight of an ordinary lathed and plastered ceiling is about 10 lbs. per square foot; and that of an ordinary floor of 1¼ inch boards, together with the usual 3 x 12 inch joists, 15 inches apart from centre to centre, is from 10 to 12 lbs. per square foot. White

pine timber, if dry, may be considered to weigh about 25 lbs., northern yellow pine 35 lbs., and southern yellow pine 45 lbs. per cubic foot. If wet, add from 20 to 50 per cent. Oak may be reckoned at from 40 to 50 lbs. per cubic foot; cast iron at 450 lbs. per cubic foot; wrought iron at 480 lbs. per cubic foot.

The allowance to be made for the weight of snow will depend upon the latitude. It may accumulate in considerable quantities, becoming saturated with water and turning to ice. The weight of a cubic foot is very various. Freshly fallen snow may weigh from 5 to 12 lbs. Snow and hail, sleet, or ice may weigh from 30 to 50 lbs. per cubic foot, but the quantity on a roof will usually be small. Snow saturated with water will generally slide off from roofs of ordinary pitch. An allowance of from 12 to 15 lbs. per square foot of roof will suffice for most latitudes.

After the stresses on the frame are determined, the several parts must be designed to withstand them. It is not the purpose to make here any extended remarks on the way to work up a design and get out the details, as we wished simply to explain the Graphic Method. A few suggestions however, in conclusion, in regard to the proper proportioning of the different members may not be amiss.

As materials, if repeatedly strained to an amount at all approaching the breaking strain, will fail sooner or later, the severe action weakening them, and as we must provide for unforeseen and unknown defects of material and workmanship, it is customary to so design a structure that it will require a load several times greater than the one to which it is to be submitted to break or injure any piece. The ratio of the breaking load to the actual or working load is called the *factor of safety*. For roofs and structures under a steady load this factor of safety is commonly 3 or 4, sometimes more. For structures like bridges, where the load is movable, and where they are exposed to shocks, the factor is much larger, often double in amount, so far as concerns the moving load. The working stresses on timber are smaller fractions of the breaking stress than is the case with iron. The quality of the iron employed materially affects the force which it may safely resist.

Pieces in tension will be liable to break at the smallest cross-section. It is therefore economical of material to enlarge the screw ends of iron rods and bolts so that the cross-section at the bottom of

the threads shall be at least as large as at any other point. It is desirable that the centre of resistance of the cross-section of struts and ties shall coincide with the centre of figure, as a deviation from that point weakens the piece. To calculate the net or smallest cross-section of a tension member it is sufficient to divide the force to be resisted by the safe working stress, or to multiply the force by the factor of safety and divide by the breaking strength per square inch. Let P=the force to be resisted in pounds, s=the factor of safety, f=the breaking strength in pounds per square inch of section, and A=the cross section in square inches; then

$$A = \frac{P s}{f}.$$

The value of f for tension may be taken as—for wood, 10,000 to 12,000 lbs.; for wrought iron bars and bolts of small dimensions and excellent quality, 60,000 lbs.; plates and bars of ordinary quality, 45,000 to 50,000 lbs.

For very short pieces in compression, whose lengths do not exceed six times the least dimension, the same formula may be used. As the length increases the piece has a tendency to yield sideways when compressed, and the cross-scetion must be increased. The most comprehensive formula for such pieces is that known as "Gordon's Formula." Letting l denote the length of the piece in inches, h its *least* external diameter in inches, and a a certain constant, while the remainder of the notation is the same as above, this formula, for pieces with flat, securely bedded ends, may be written

$$P s = \frac{fA}{1 + a\frac{l^2}{h^2}}.$$

The following are the values of f and a:

	f	a
Wrought iron (rectangular struts);	36,000	$\frac{1}{3000}$
Cast iron (cylindrical struts);	80,000	$\frac{1}{800}$
Timber (rectangular struts);	7,200	$\frac{1}{250}$

If the struts are jointed at their ends, by pin connections, or are so narrow as to readily yield sideways at these points, use $4a$ in place of a. If one end is firmly fixed in direction, while the other end is jointed, use $\frac{16}{9}a$ in place of a.

It is convenient, for cross-sections other than circles, to assume h and compute A. If the other dimension then comes smaller than h, a less value must be taken for h and the calculation made anew.

For cross-sections in wrought iron not rectangles, such as **L**, **T**, and **H** sections, use $1\frac{1}{2}\, l$ in place of l.

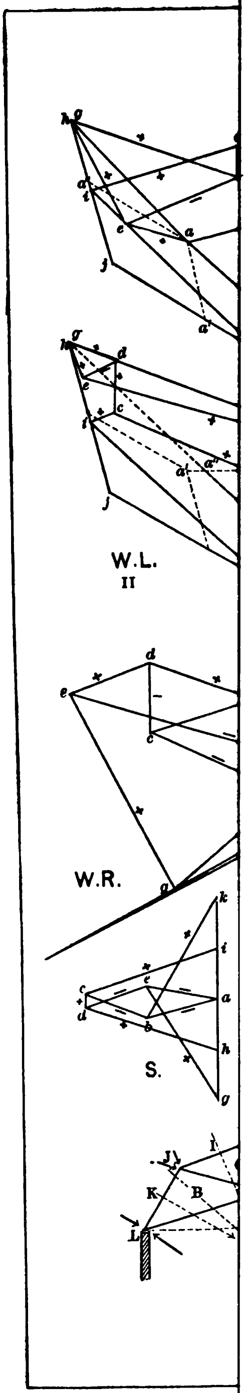
In determining the moment of resistance of a piece exposed to bending, or in calculating the cross-section required to resist the bending moment, allowance must also be made for portions cut away in attaching fastenings, etc. A curved piece, transmitting either thrust or tension, experiences also a bending moment, as stated earlier, equal to the direct stress transmitted between its ends, multiplied by the perpendicular from the line joining its two ends to the centre line of the piece. Those pieces which resist both a bending moment and a direct stress may first be designed to safely carry the bending moment, and then the dimension transverse to that in which the piece will bend may be so much increased that the added slice will resist the direct pull or thrust. If that force is thrust it will be well to test the size of the piece by Gordon's Formula also.

To rightly proportion the eyes and pins for the connection of tension bars is not an easy matter. The following rules from the Phœnix Iron Co.'s Pocket Book will be found serviceable:—

"Make the diameter of the pin from $\frac{3}{4}$ to $\frac{4}{5}$ of the width of the bar in flats, and $1\frac{1}{4}$ times the diameter of the bar in rounds, giving the eye a sectional area of 50 per cent. in excess of that of the bar. The thickness of flat bars should be at least one-fourth of the width in order to secure a good bearing surface on the pin, and the metal at the eyes should be as thick as the bars." Many other useful details are given in the same book.

Most of the pocket and hand books issued by the different iron companies, for the use of their patrons, give the sections and weights of the various shapes of rolled bars, the usual lengths, the safe loads for beams of different spans, etc.

Very close attention must be given to all minor details; to so proportion all the parts of a joint that it will be no more likely to yield in one way than another; to weaken as little as possible the pieces connected at a splice; to give sufficient bearing surface so as to bring the intensity of stress on the surface within proper limits; to distribute rivets and bolts so as to give the greatest resistance with the least cutting away of other parts. In short, a failure of a joint or connection is as fatal to a frame as to have a member too small for the stress upon it.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